Robust Non-Interactive Zero-Knowledge Combiners

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Hong Kong Polytechnic University

EUROCRYPT 2026

Bird’s eye view

Warm-up: Robust Combiners	Definitions & Impossibility Results
Constructions	Wrapping up

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Robust Combiners

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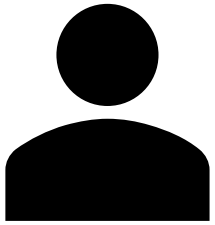
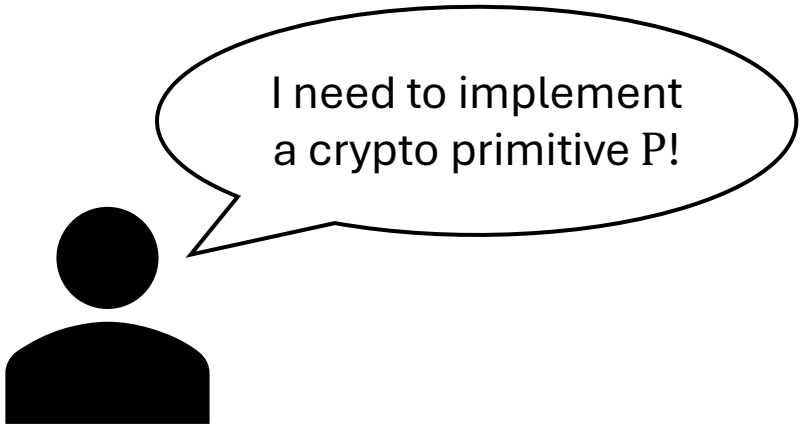


Alice

Robust Combiners



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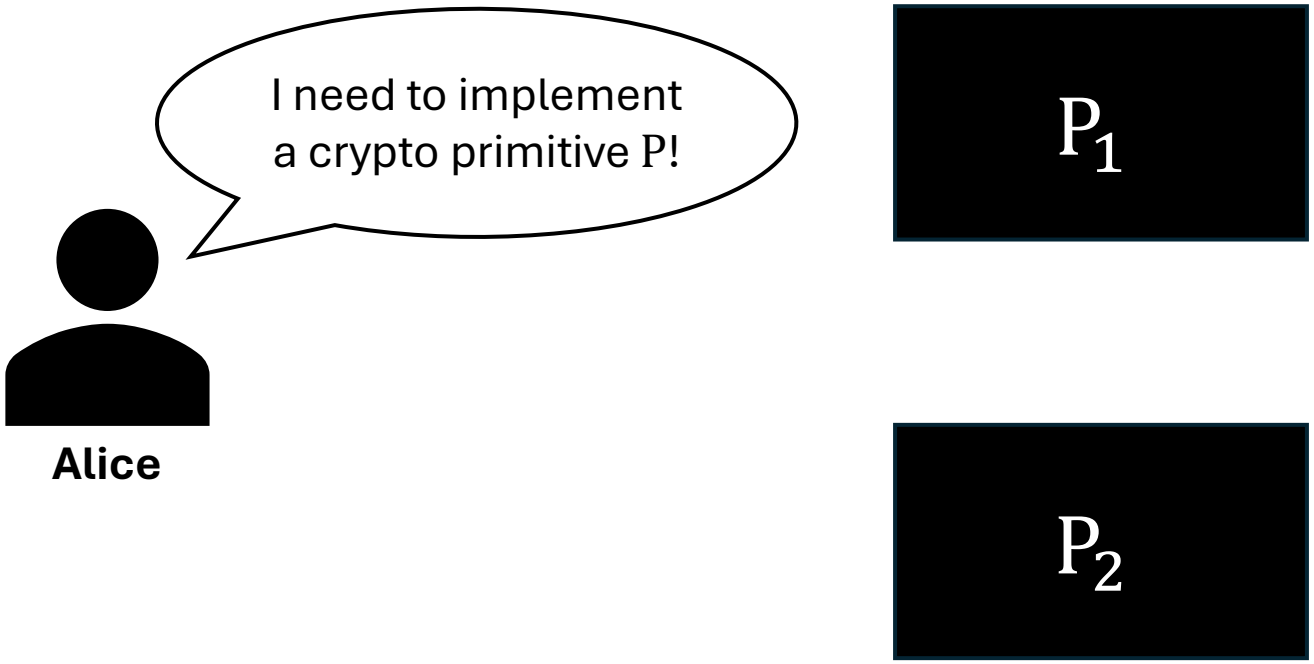

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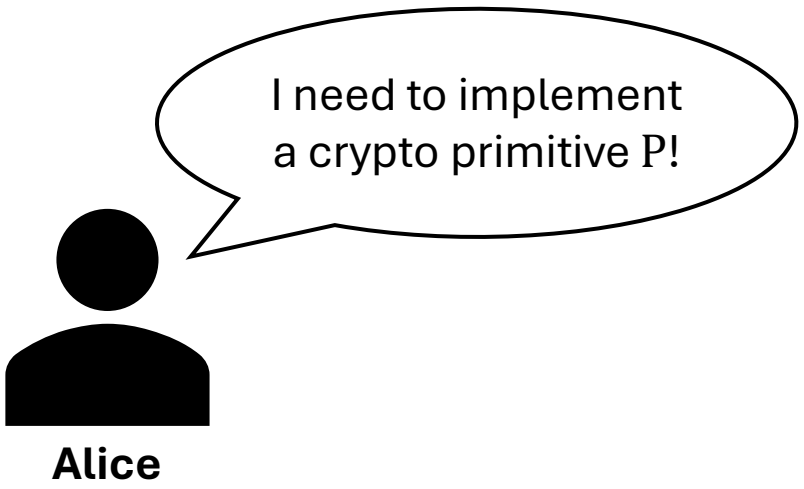
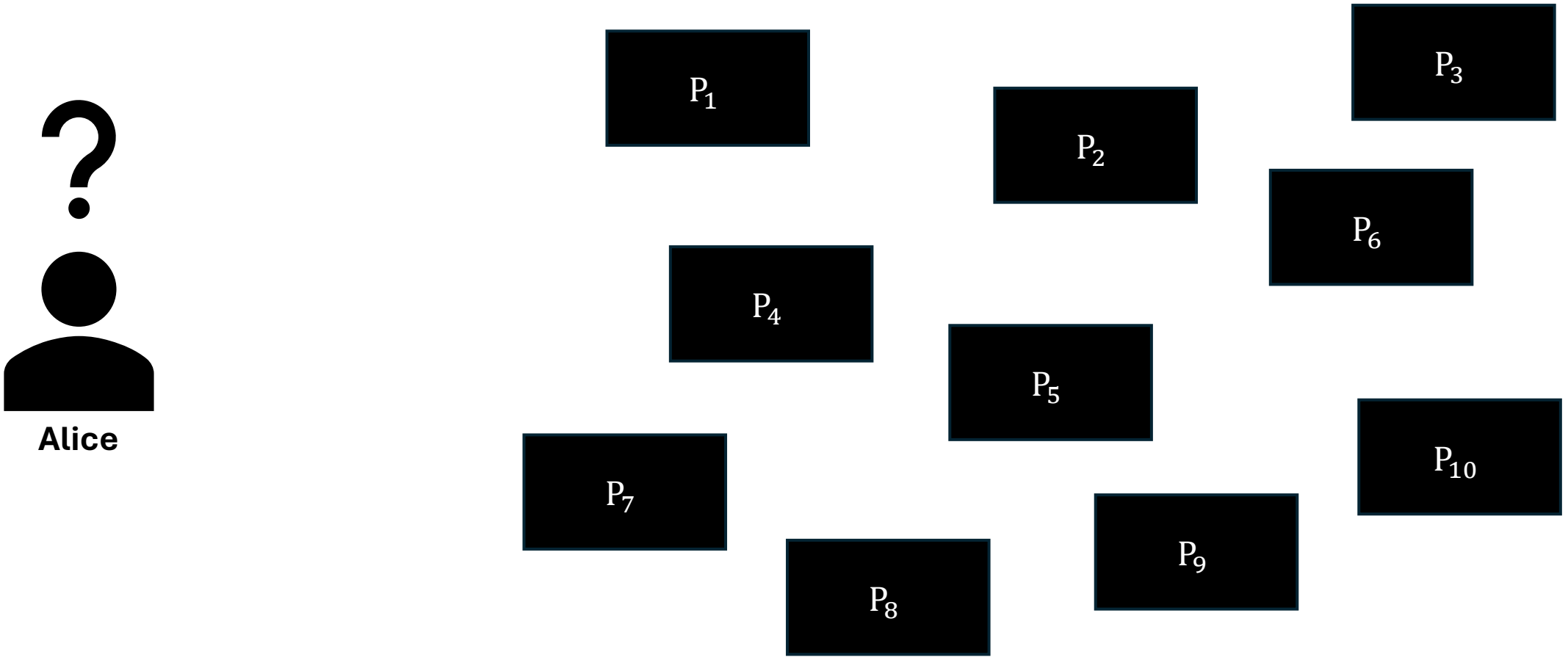


P_1

P_2

- Pre-quantum vs post-quantum?
- Setup assumptions?
- Open vs closed source?
- …

Robust Combiners



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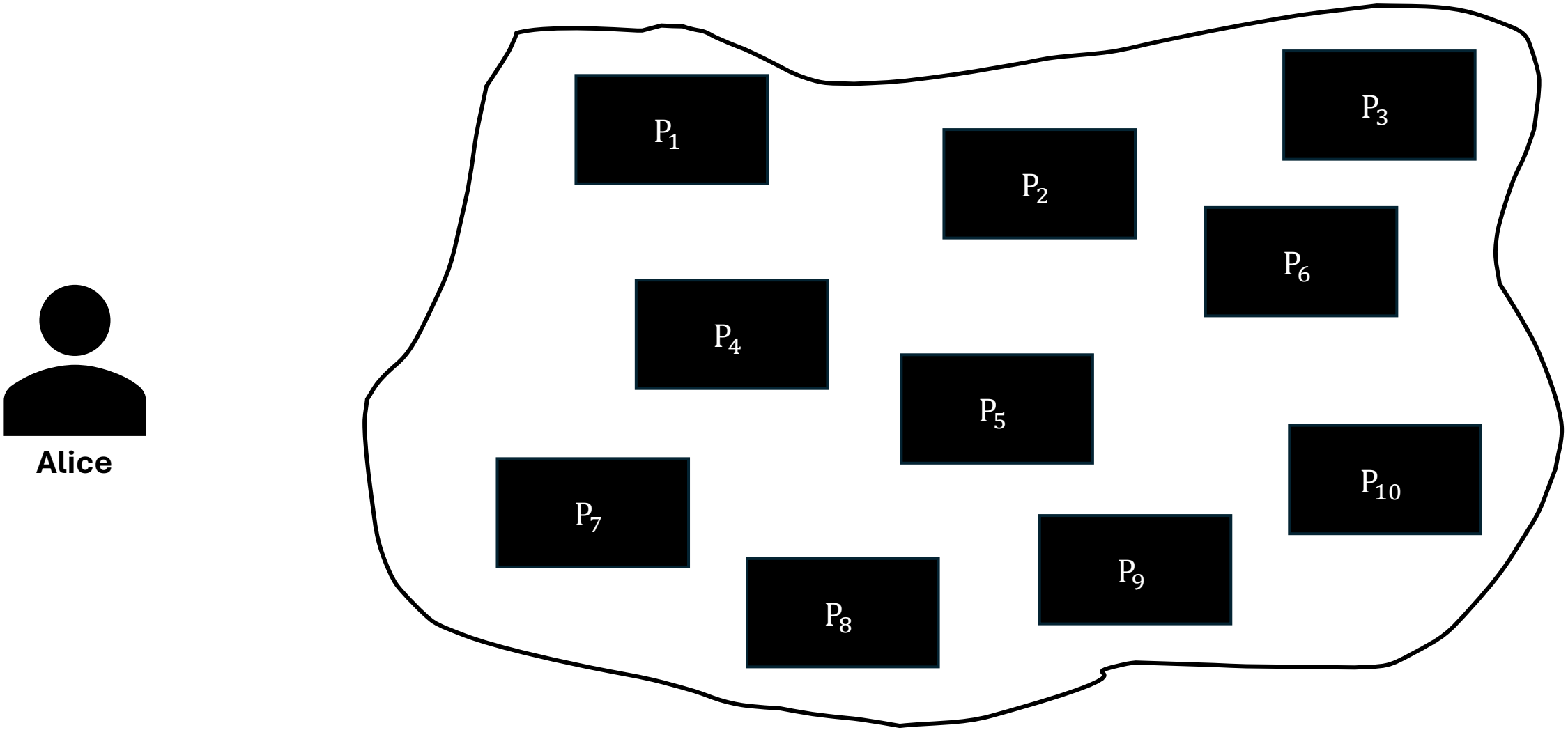

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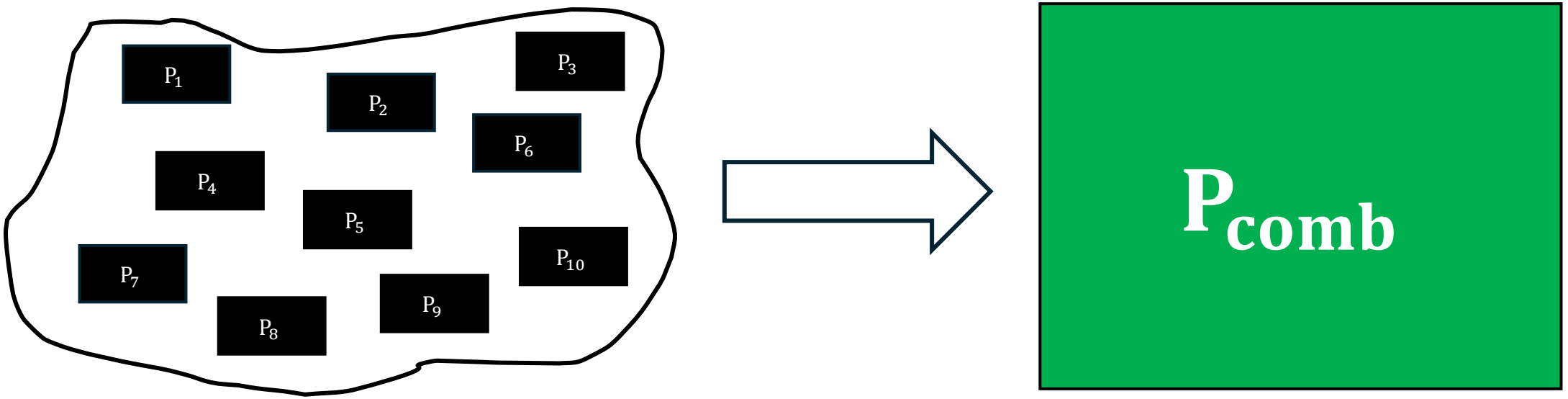

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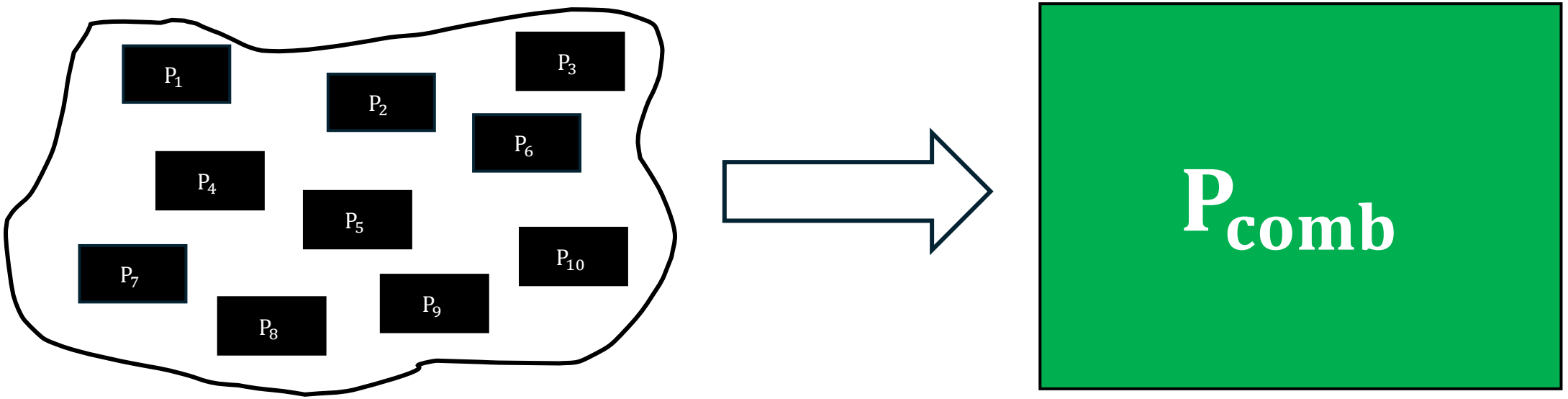

- "As long as at least t out of the n candidates are secure (with $t \leq n$), the combiner is also secure".

[HKN+05] D. Harnik, J. Kilian, M. Naor, O. Reingold, and A. Rosen, “On Robust Combiners for Oblivious Transfer and Other Primitives.” EUROCRYPT’05. https://www.iacr.org/archive/eurocrypt2005/34940096/34940096.pdf

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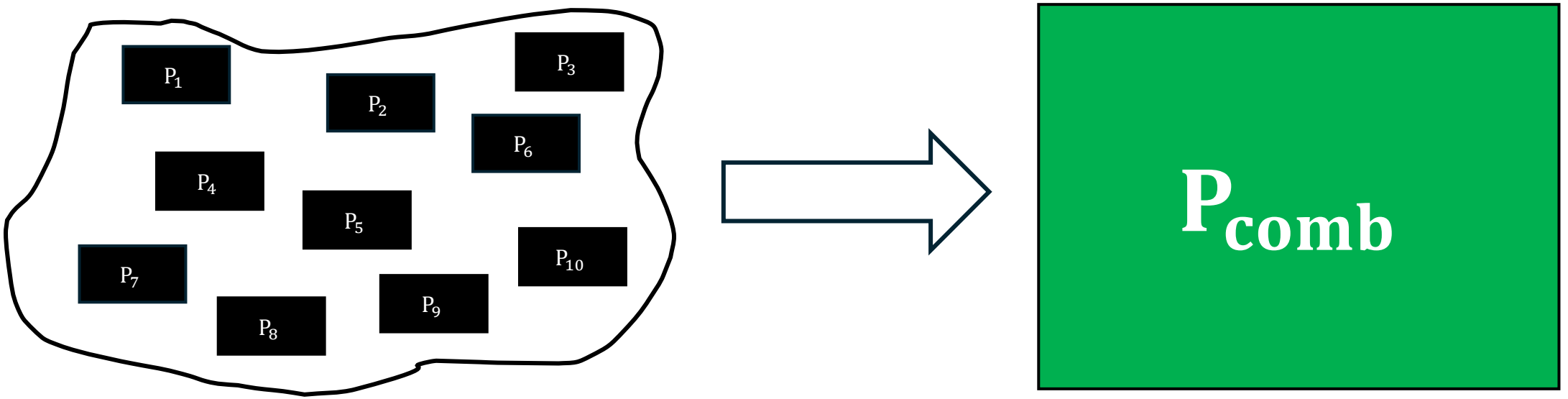

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  - **Robustness**: the combiner’s security **must** derive from its candidates.

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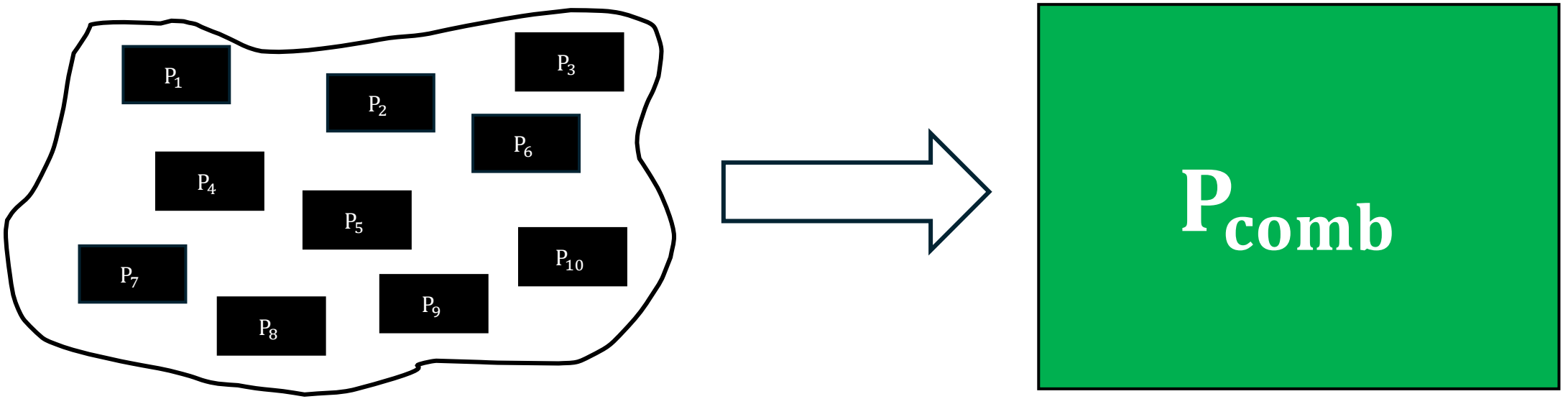

- “As long as at least t out of the n candidates are secure (with $t \leq n$), the combiner is also secure”.
  - **Robustness**: the combiner’s security **must** derive from its candidates.
- **More formally**: if the combiner is broken, we have at least $(n - t) + 1$ reductions to the candidates.

A simple example: CRH!

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$$\mathrm{H}_{\mathrm{comb}}(\mathrm{x}) = \mathrm{H}_1(\mathrm{x}) \,||\, \mathrm{H}_2(\mathrm{x}) \,||\, \mathrm{H}_3(\mathrm{x})$$

A simple example: CRH!

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- Optimal security: **one** good candidate suffices.

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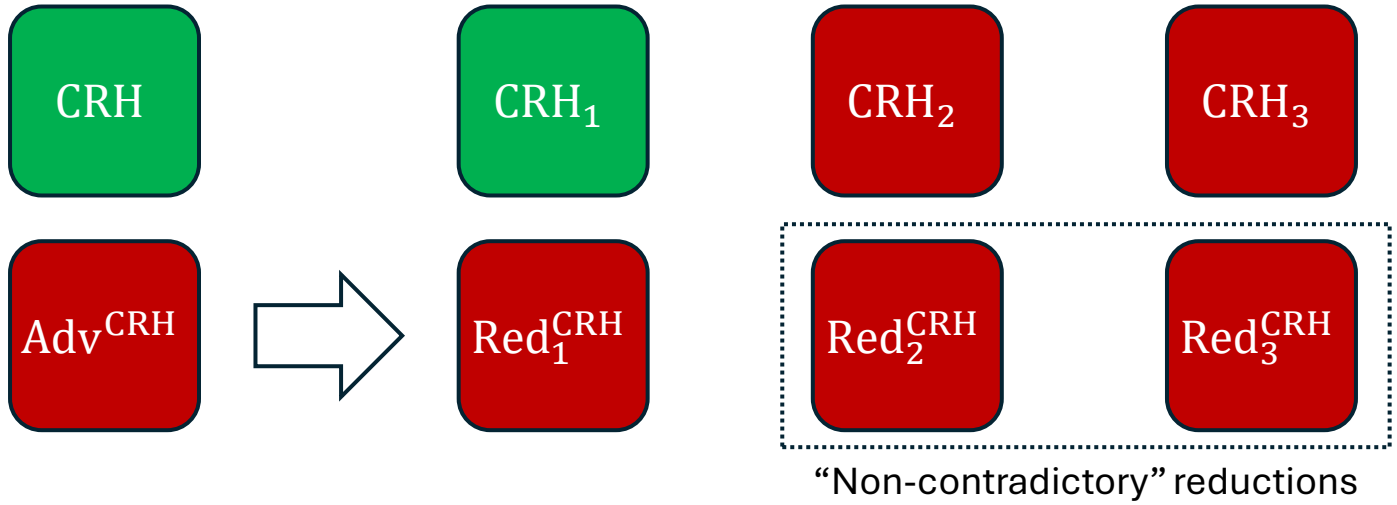

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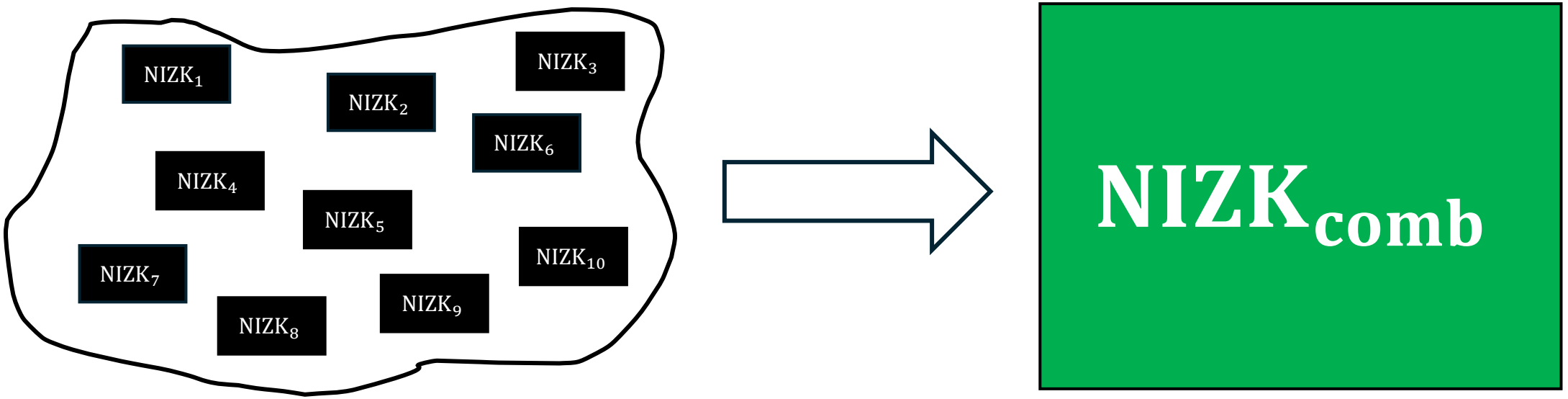
Robust NIZK Combiners



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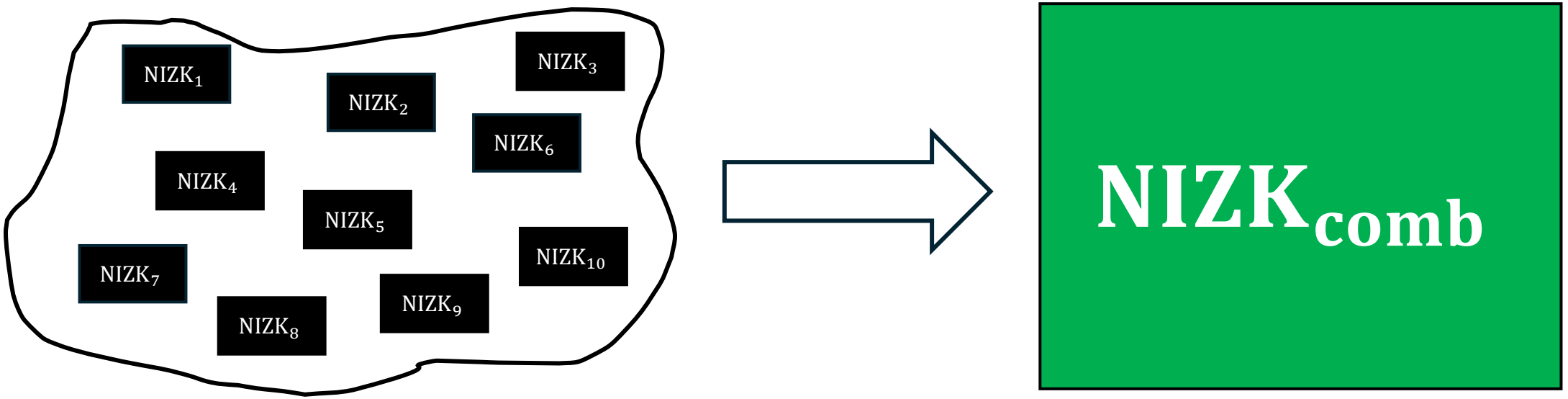
Robust NIZK Combiners



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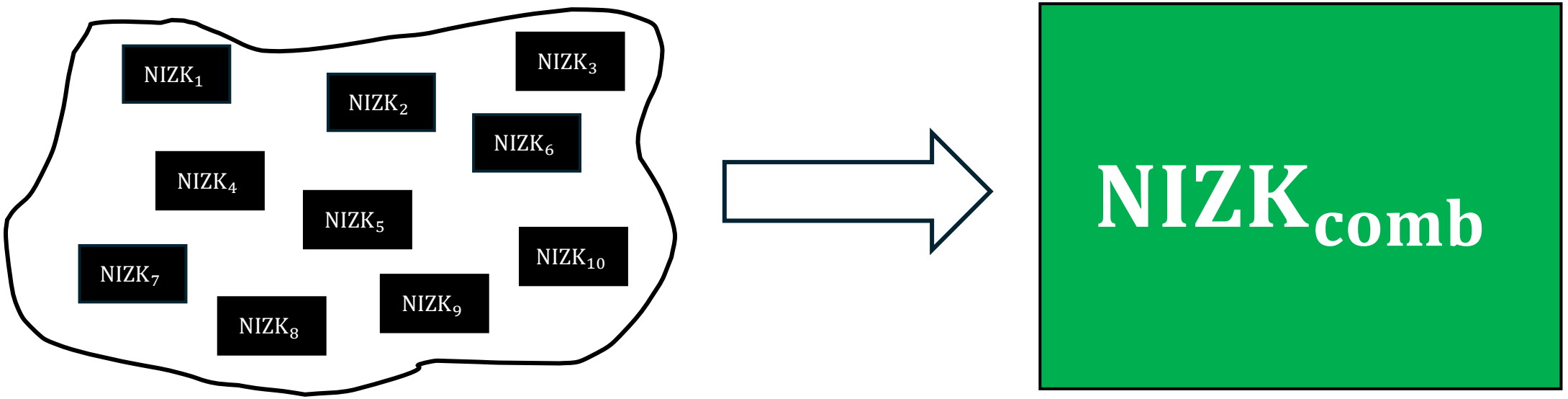
  - NIZK is **multi-property**.

Robust NIZK Combiners



- "As long as at least t out of the n candidates are secure (with $t \leq n$), the combiner is also secure".
  - NIZK is **multi-property**.
  - What does t mean? What are we breaking?

(Sound, ZK)-Robust Combiner

$n = 3$	$t = 2$
$(n - t) + 1 = 2$	

$$\mathrm{NIZK}_{\mathrm{comb}} = f(\mathrm{NIZK}_1, \mathrm{NIZK}_2, \mathrm{NIZK}_3)$$

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ZK

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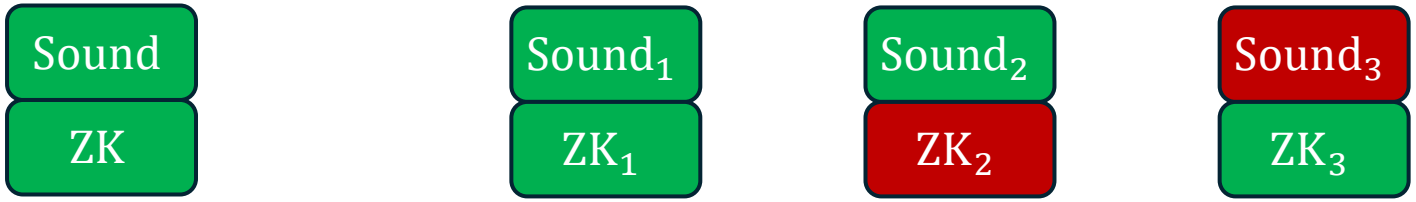

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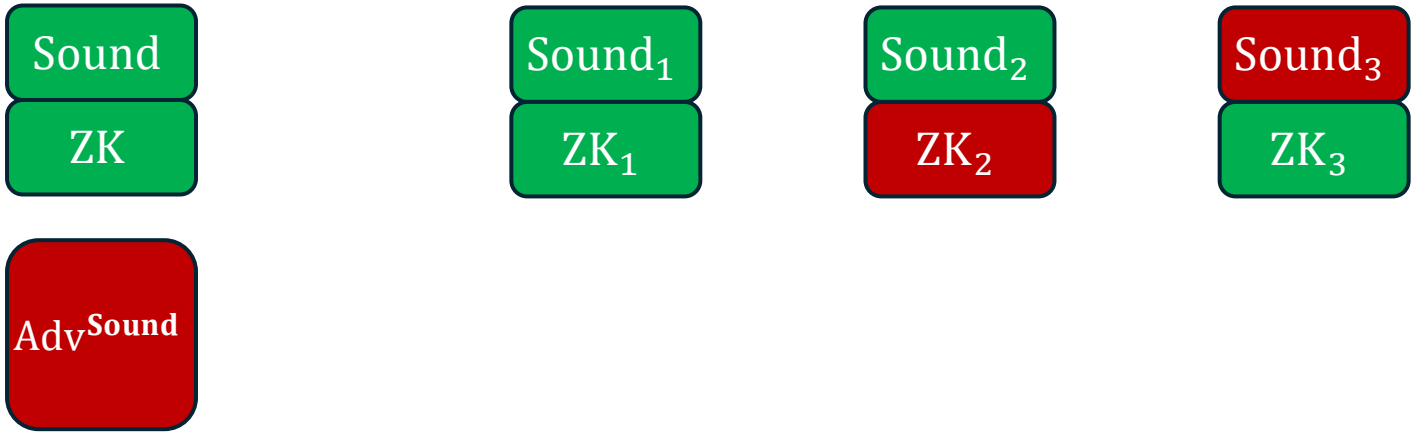

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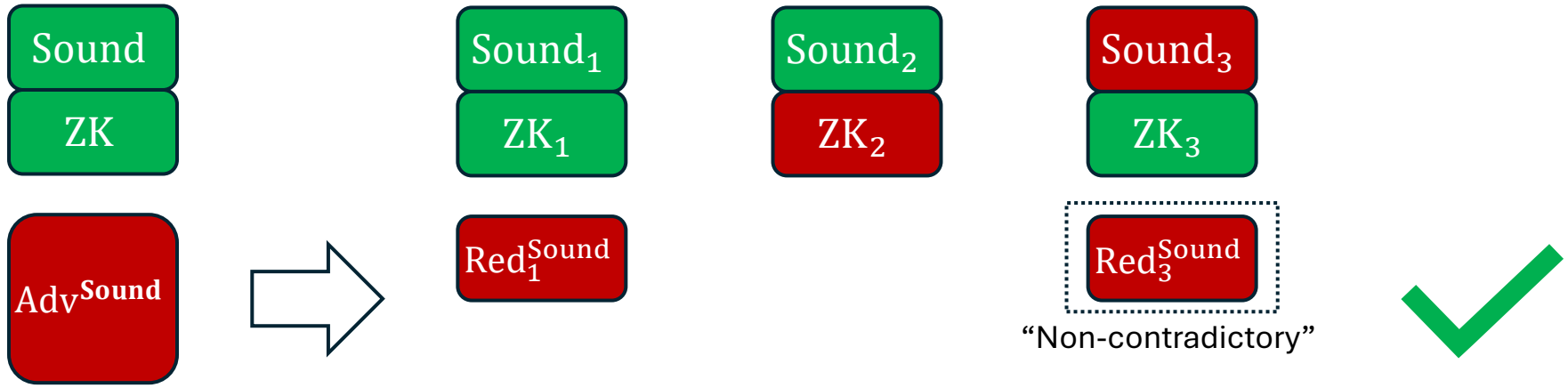

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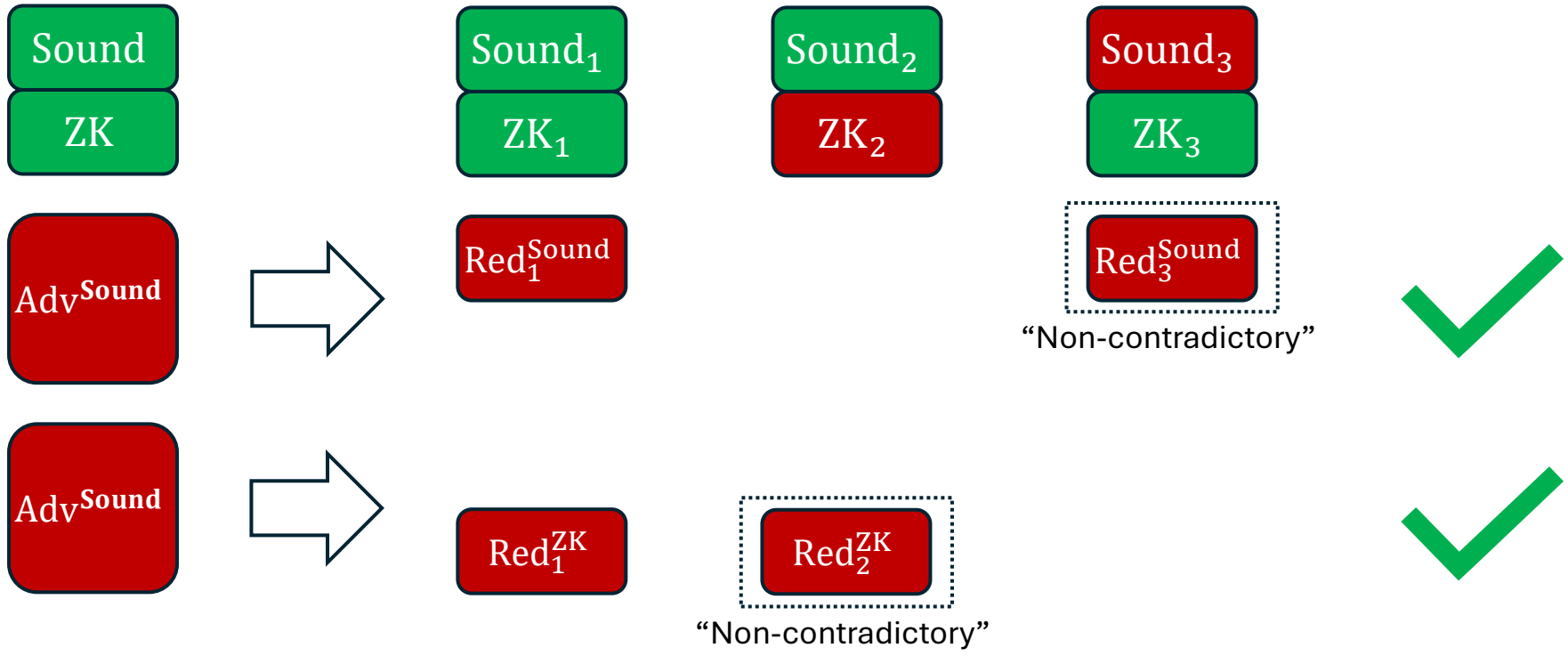

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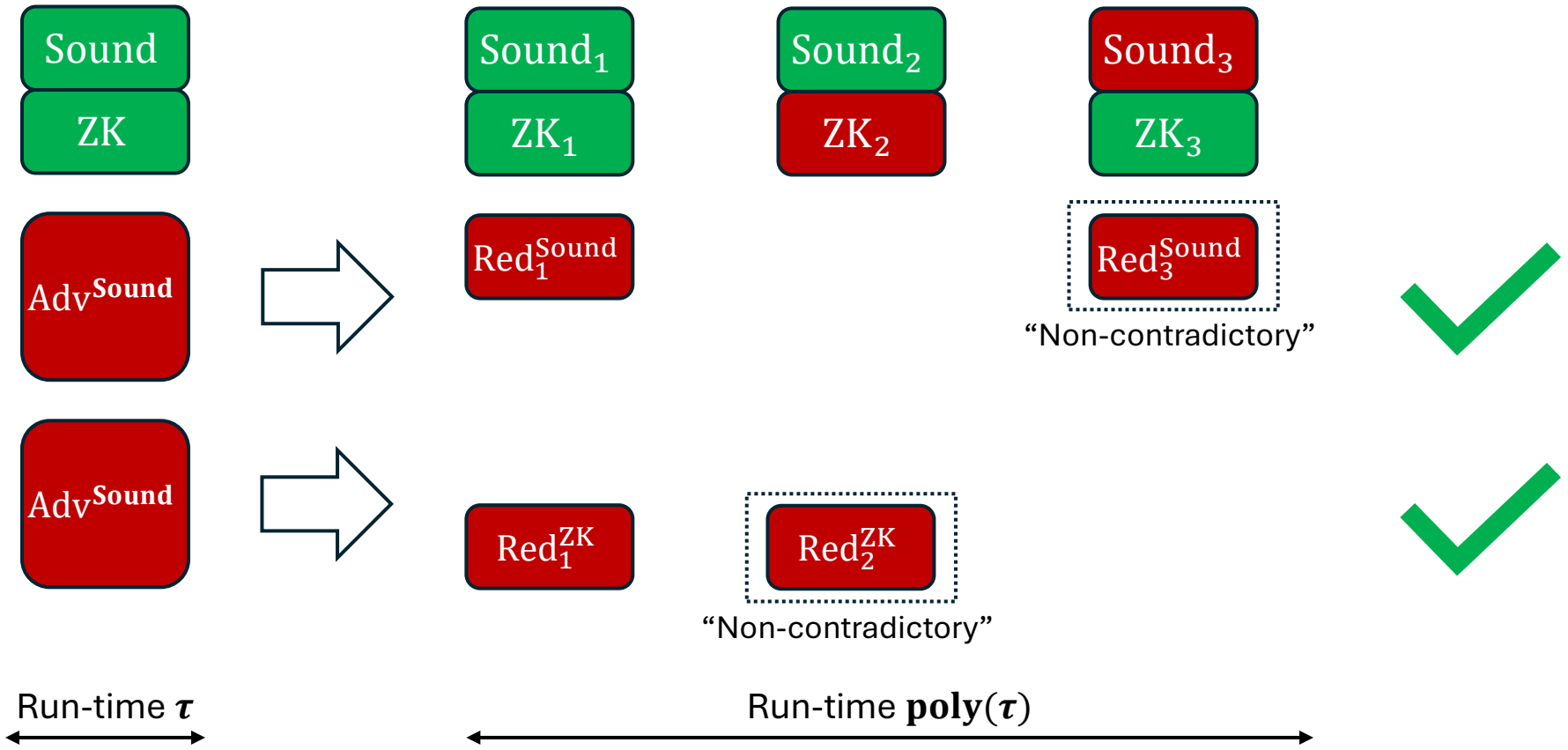

Impossibility for $t \leq n/2$

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- Idea: exhibit a **counterexample**!

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NIZK_1

- $\mathrm{Setup}(1^\lambda)$: Output $\perp$.
- $\mathrm{P}(x, w)$: Output w.
- $\mathrm{V}(x, \pi)$: Check w against x.

NIZK_2

- $\mathrm{Setup}(1^\lambda)$: Output $\perp$.
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➢ Idea: exhibit a **counterexample**!

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➢ Idea: exhibit a **counterexample**!

➢ We also adapt this counterexample to more restrictive definitions of robust combiners.

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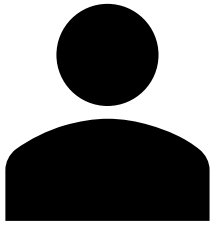
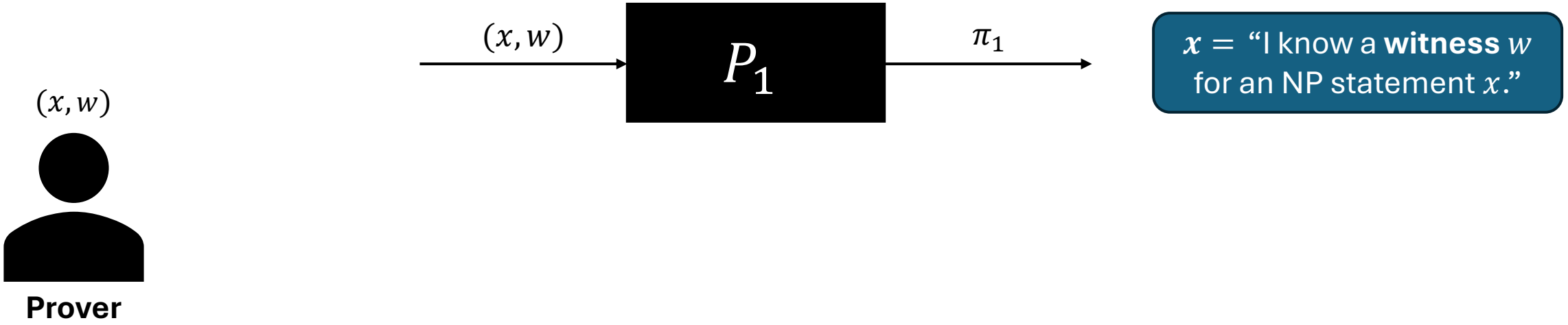
Recursive NIZK

(x, w)



Prover

Recursive NIZK



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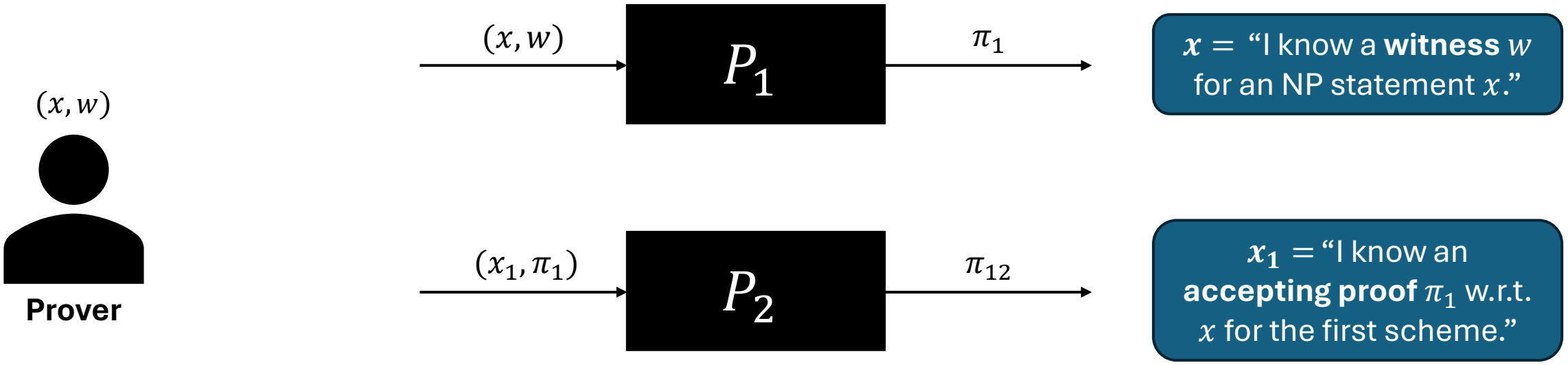

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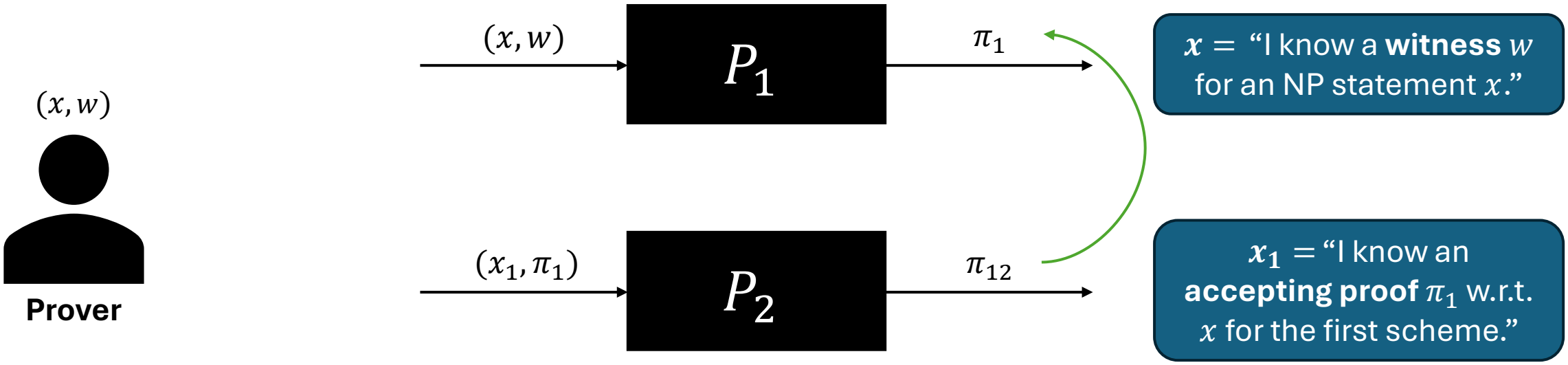

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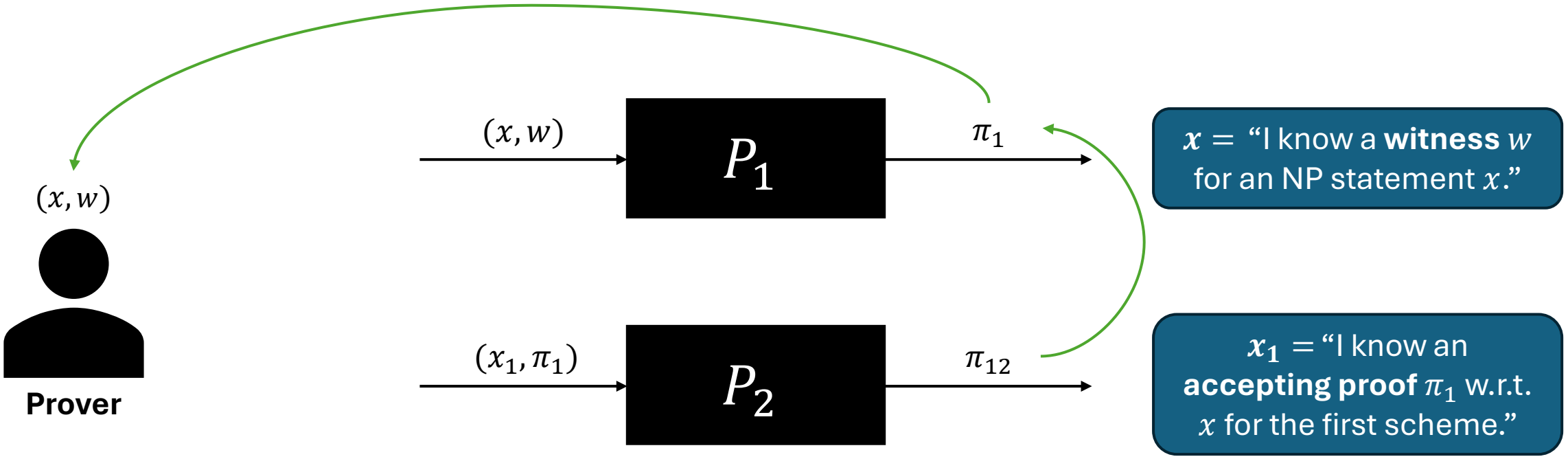
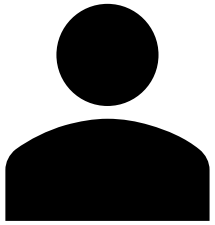


- **Core idea**: hiding π_1 while preserving knowledge soundness.

Robust combiner from recursive NIZK

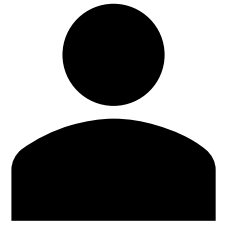
$n = 3$	$t = 2$
$(n - t) + 1 = 2$	

(x, w)



Prover

x



Verifier

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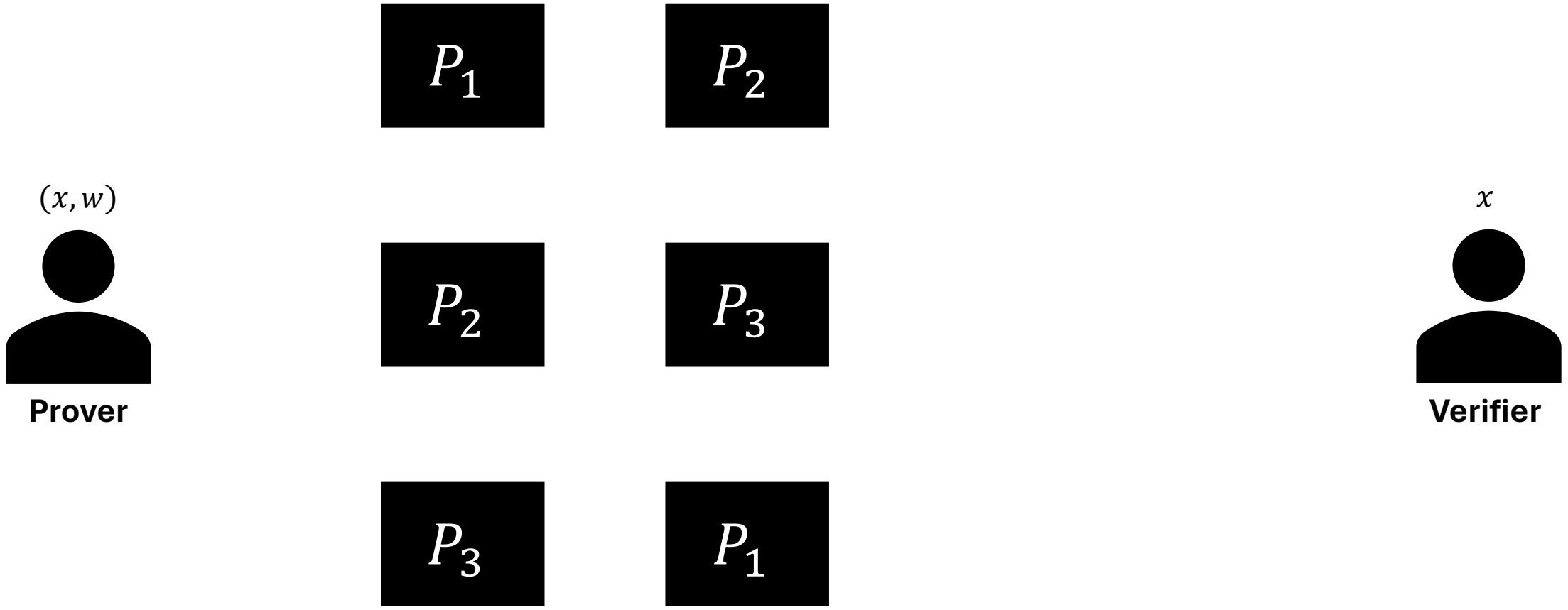

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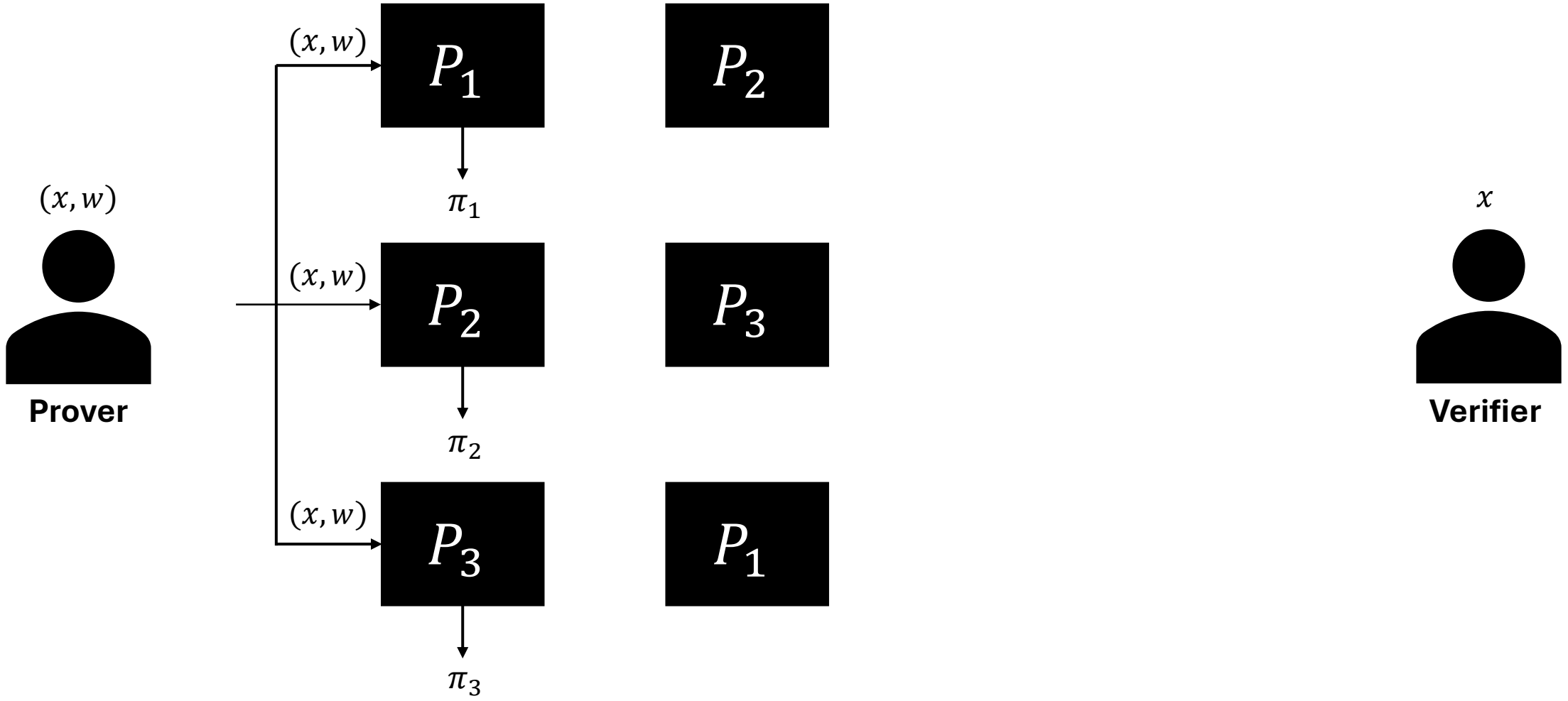

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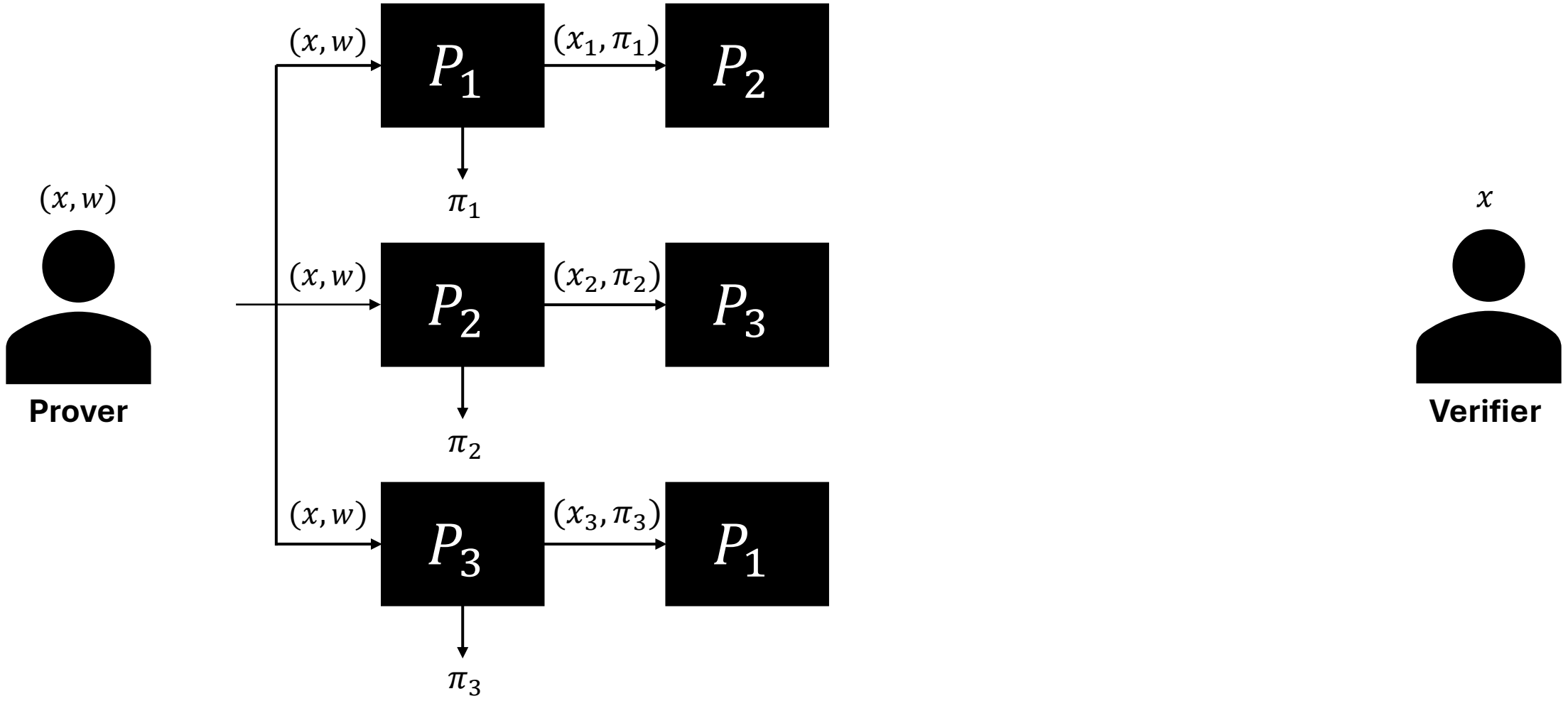

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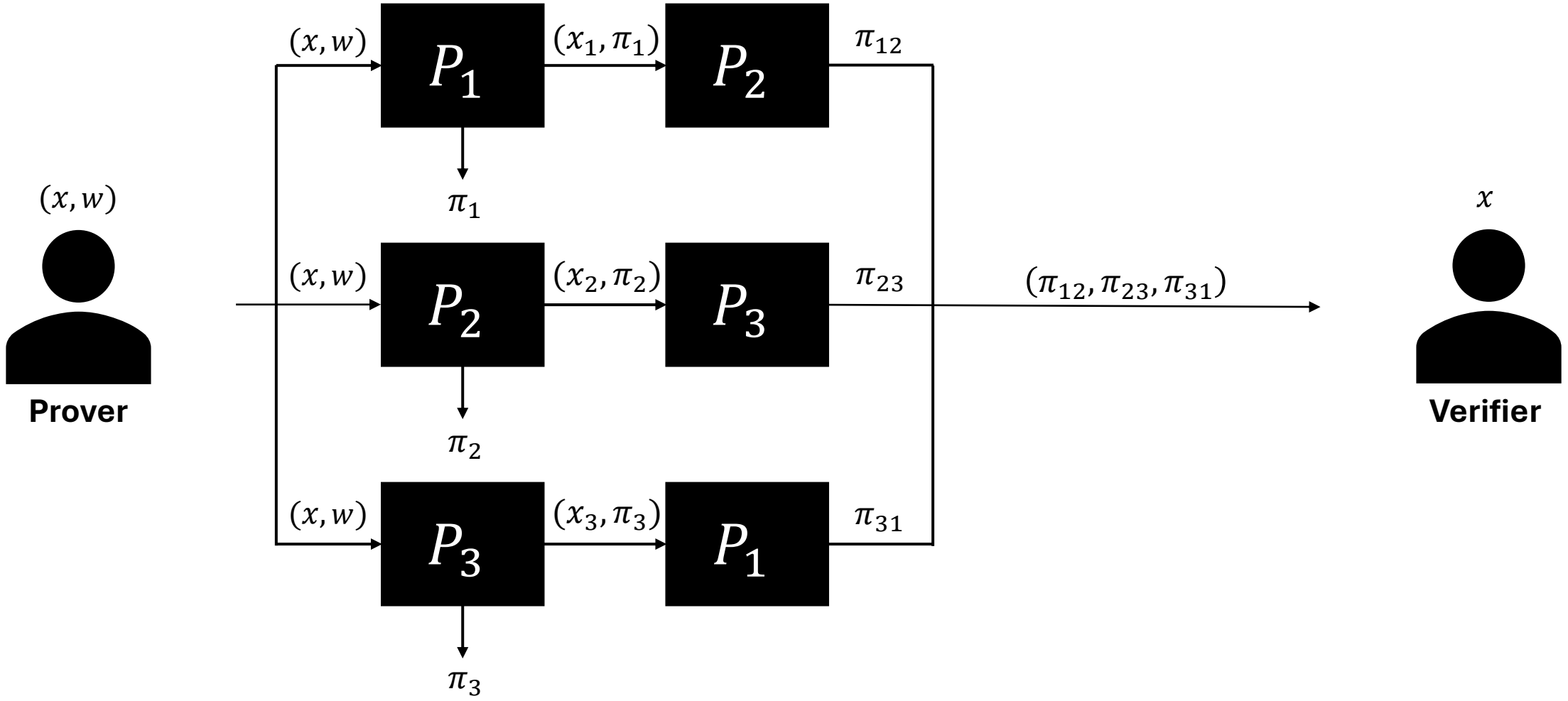

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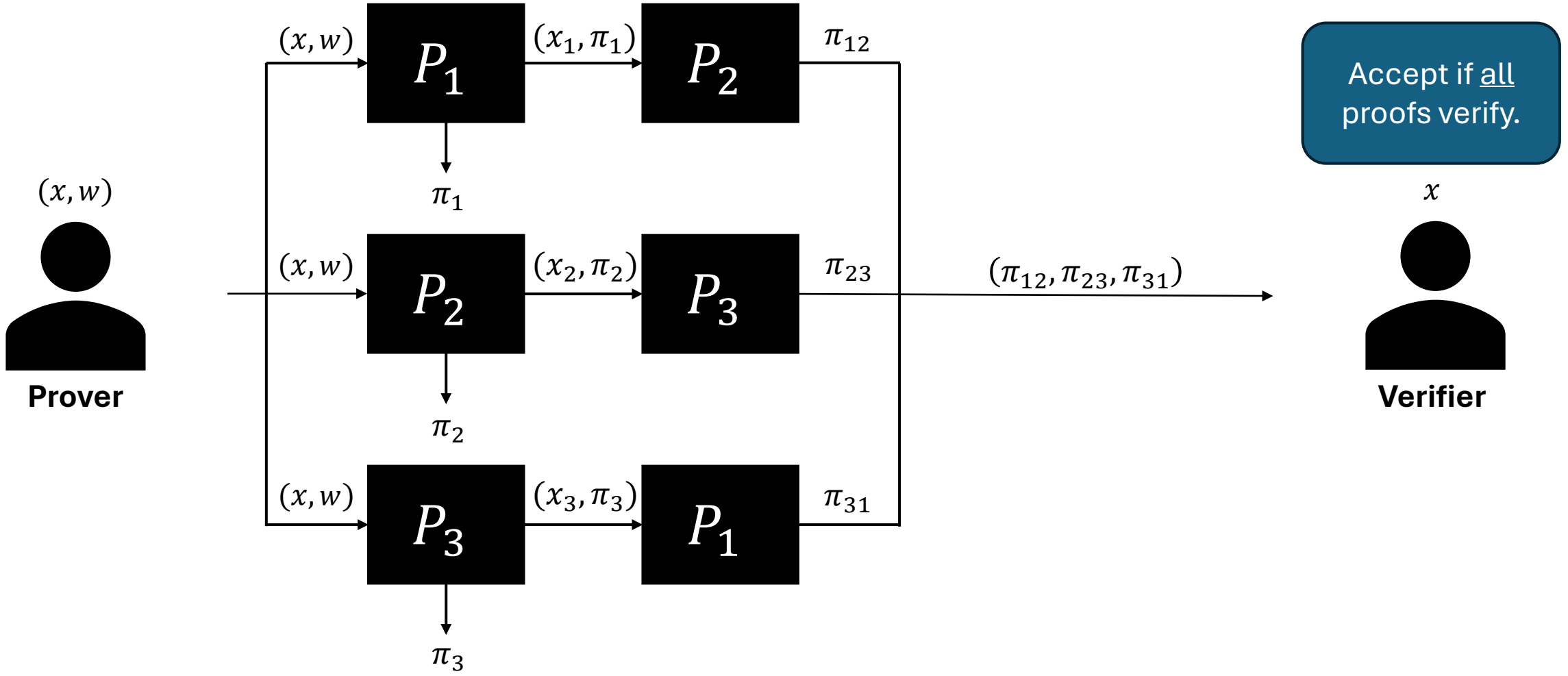
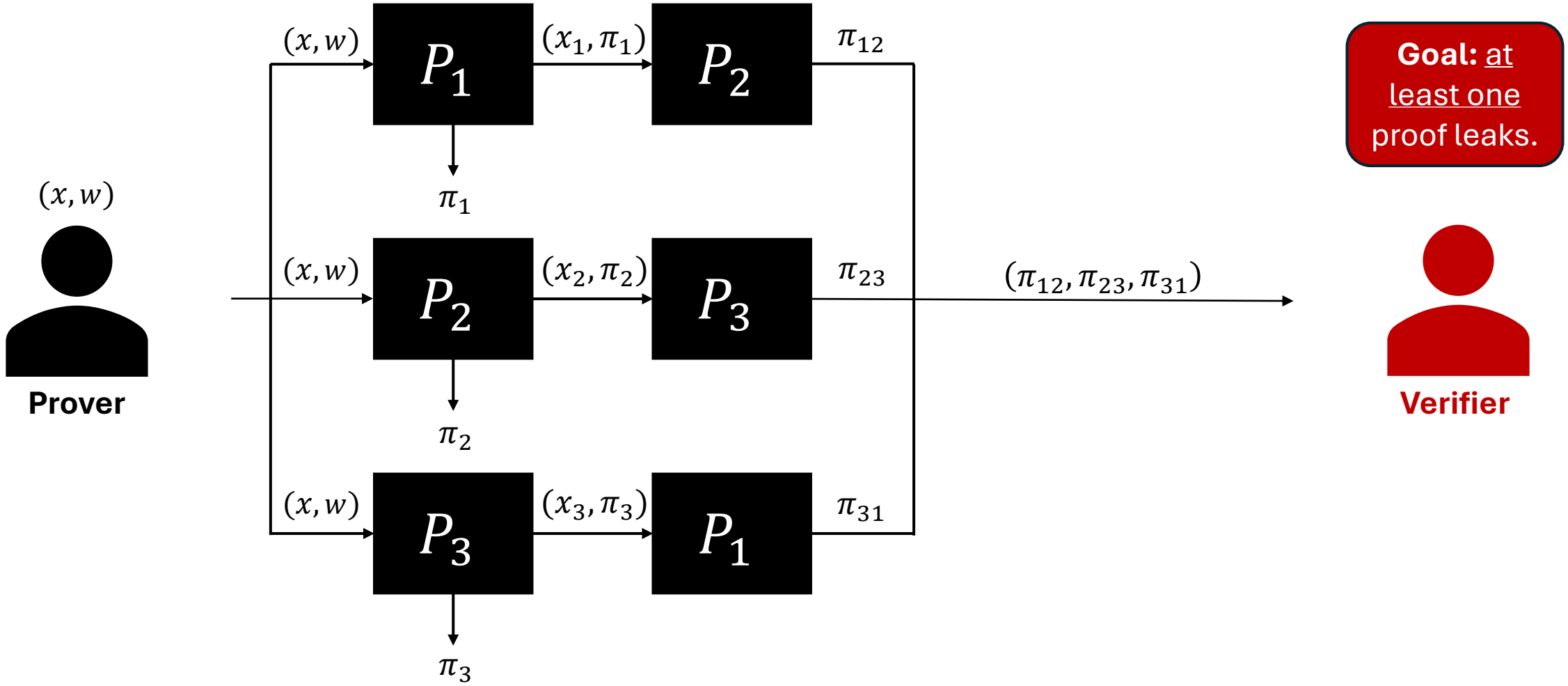
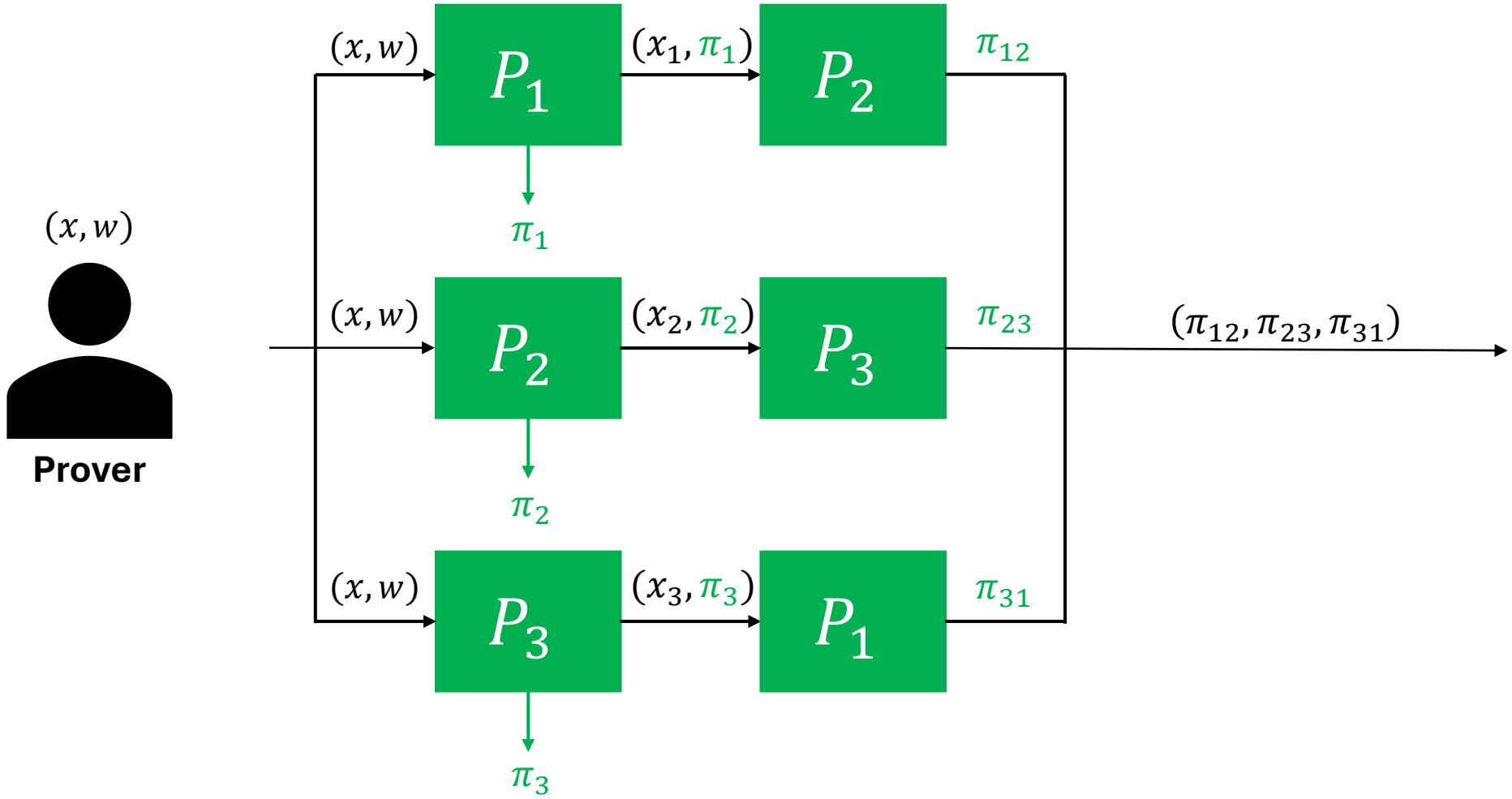


Zero-knowledge



Zero-knowledge

Color Coding
Green: no info on w.
Red: leaks w.



Goal: at least one proof leaks.

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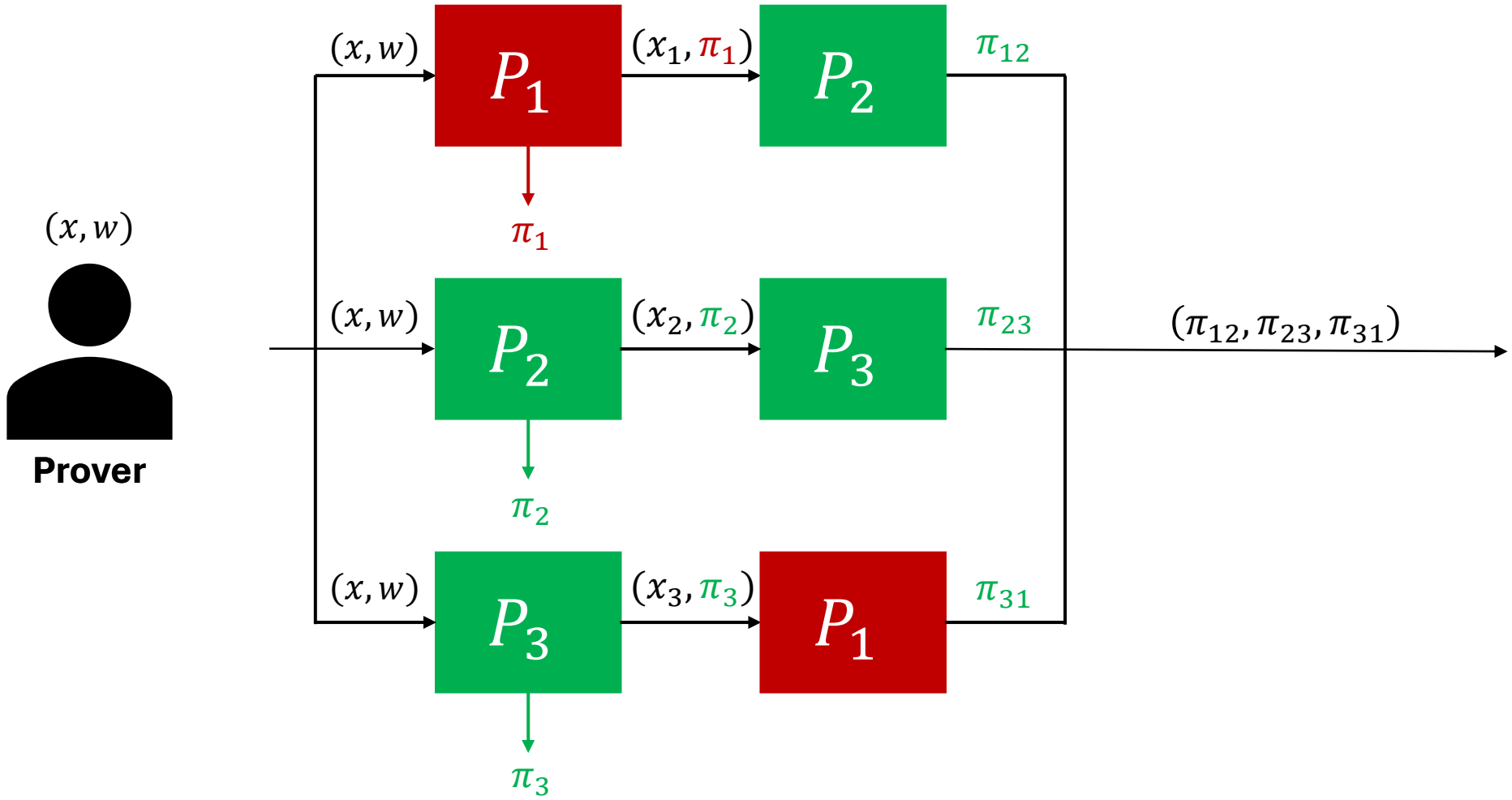

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Verifier

Red_1^{ZK}

Zero-knowledge

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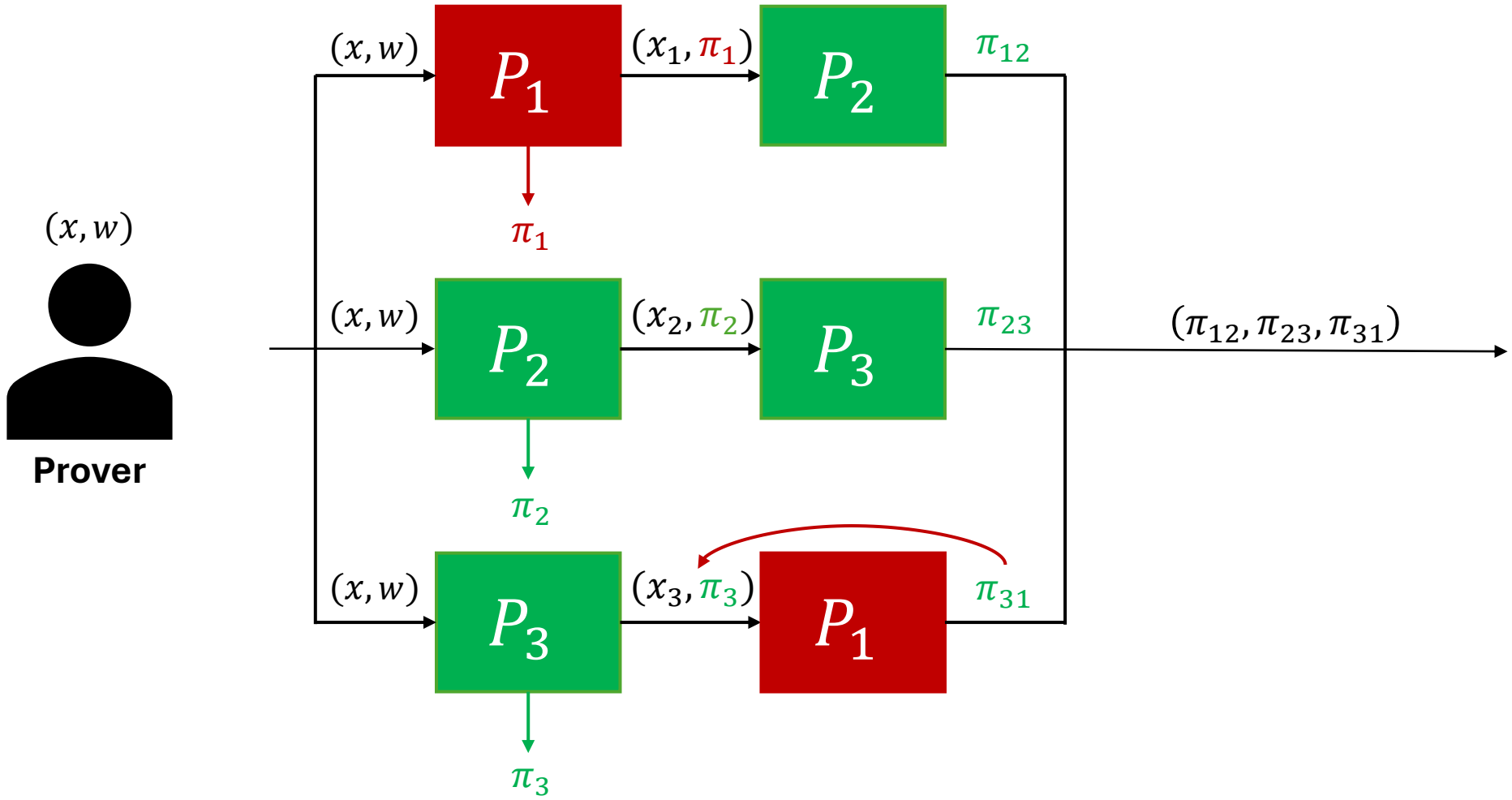

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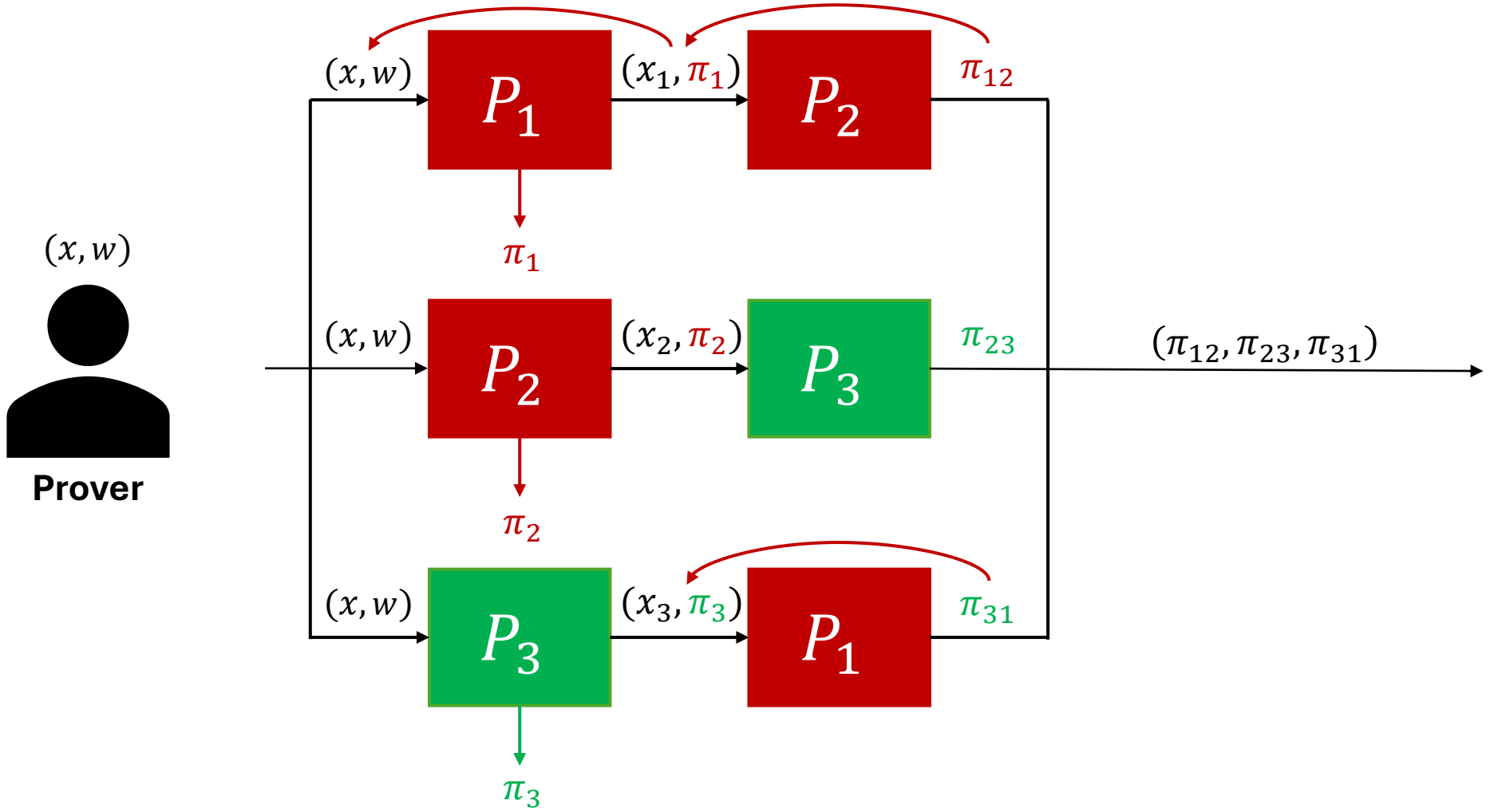

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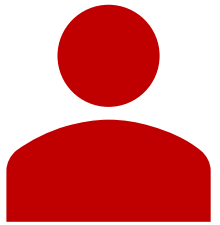

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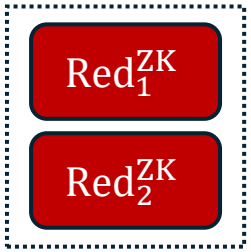

$(\boldsymbol{n} - \boldsymbol{t}) + \mathbf{1} = 2$ reductions



Constructions overview

Construction	Supported Languages	Threshold	Number of Candidates
“Recursive Proofs”	NP	$t > \lfloor n/2 \rfloor$	$n, t \in \mathrm{O}(1)$

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Construction	Supported Languages	Threshold	Number of Candidates
"Recursive Proofs"	NP	$t > \lfloor n/2 \rfloor$	$n, t \in O(1)$
"Homomorphic Secret Sharing"	Homomorphic Languages	$t > \lfloor n/2 \rfloor$	$n, t \in \text{poly}(\lambda)$
"MPC in the head"	NP	$t > \lfloor 2n/3 \rfloor$	$n, t \in \text{poly}(\lambda)$

Bird’s eye view

Warm-up: Robust Combiners	Definitions & Impossibility Results
Constructions	**Wrapping up**

Wrapping up

- **Full version**: https://ia.cr/2025/240.
  - Formal definitions and impossibility results.
  - Full details of the three constructions.
  - Security proofs (**non-trivial techniques** to find reductions).



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  - Definitions in alternative models (ROM, separate threshold for each property, …).
  - Alternative security properties (witness indistinguishability, simulation soundness, …).
  - **More constructions!**



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  - Definitions in alternative models (ROM, separate threshold for each property, …).
  - Alternative security properties (witness indistinguishability, simulation soundness, …).
  - **More constructions!**

- **Key takeaway**: combiners are yet another formal tool to distribute trust assumptions!



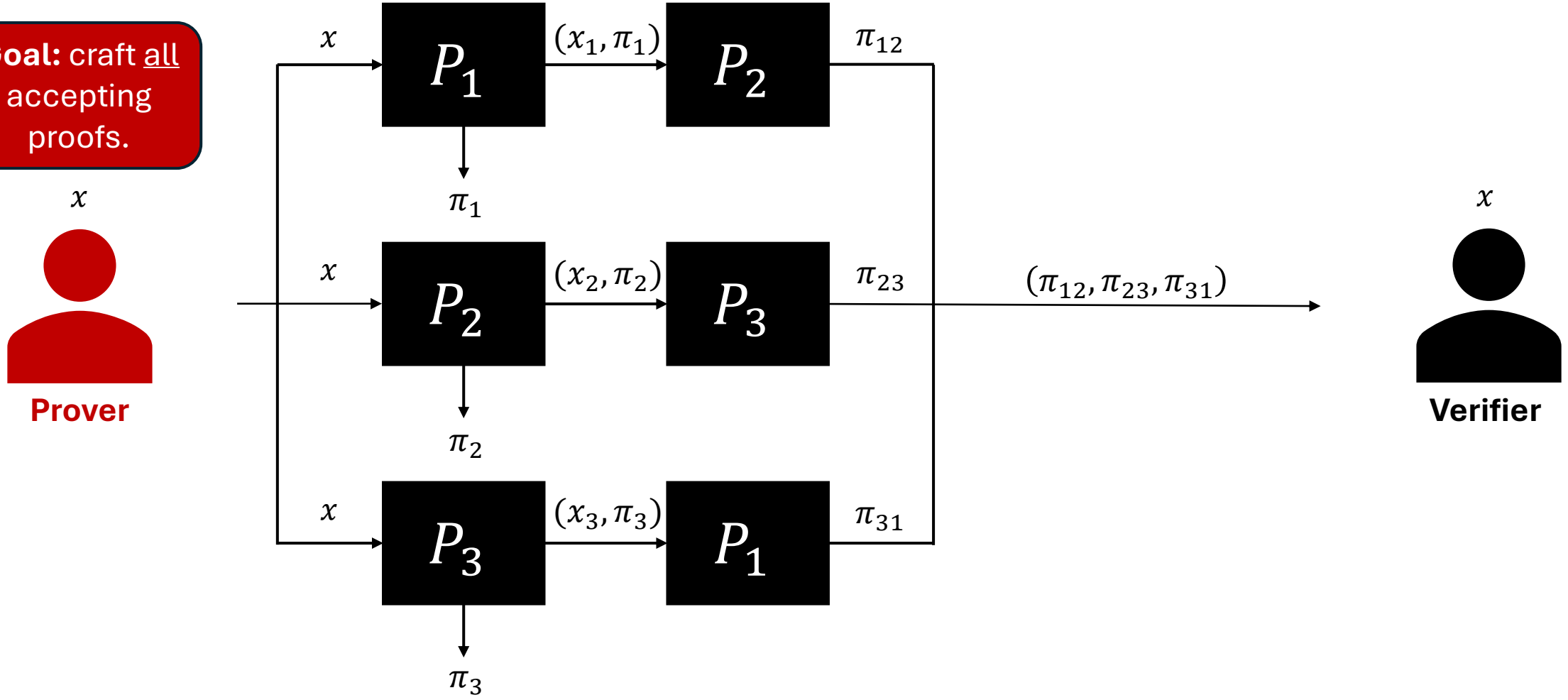
Thanks!

Backup Slides A

Knowledge soundness of the combiner from recursive NIZK

Knowledge soundness

Goal: craft <u>all</u> accepting proofs.



Knowledge soundness

Color Coding
Green: non-accepting.
Red: accepting.

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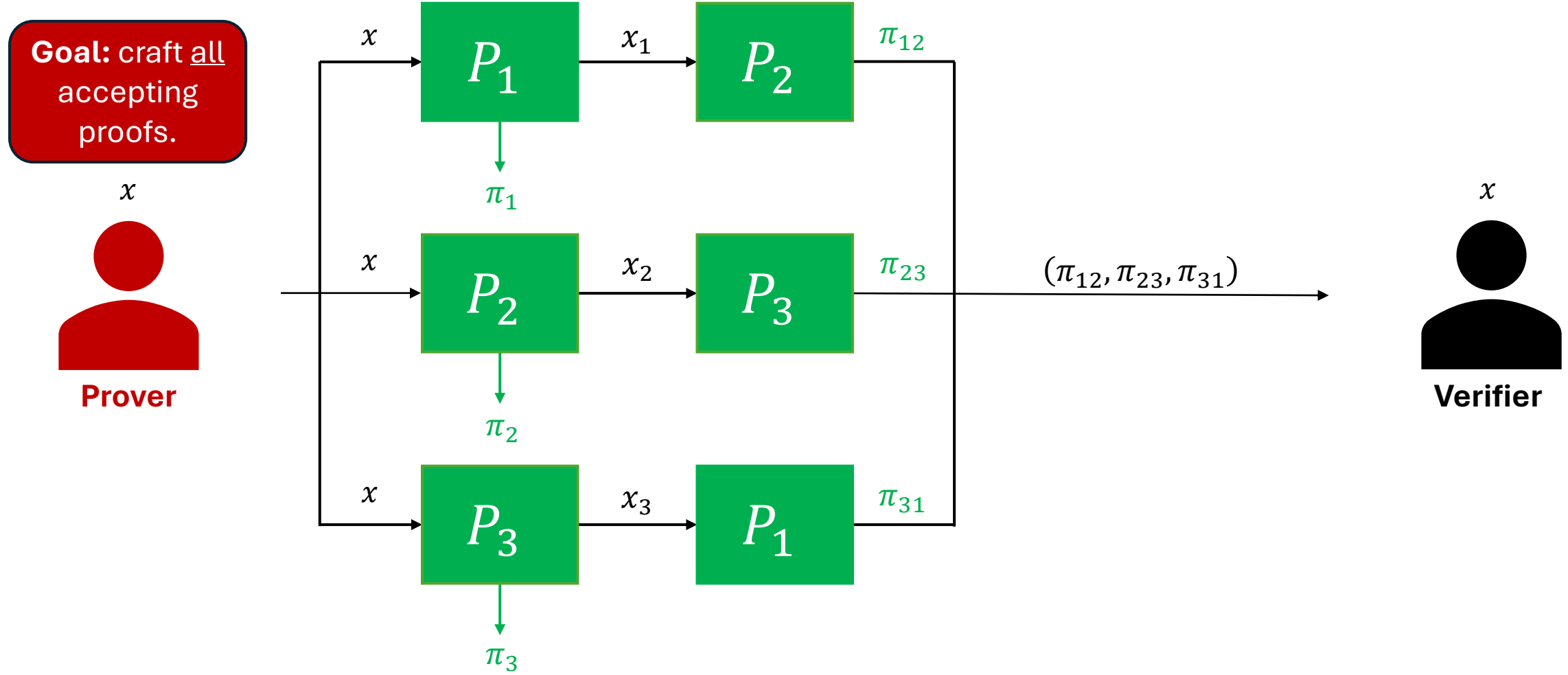

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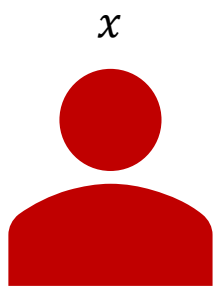
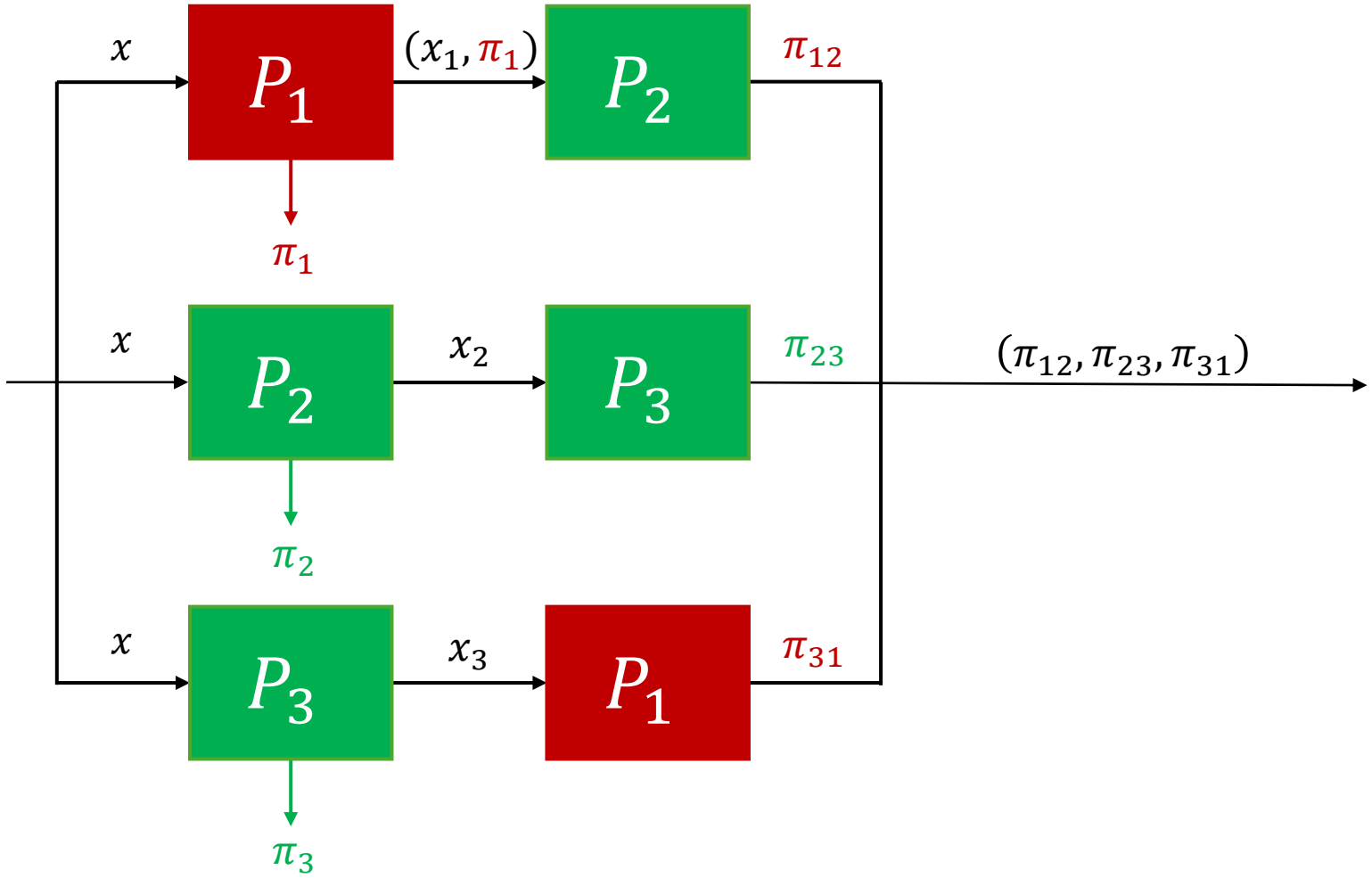
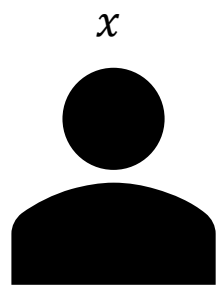


Prover

$\text{Red}_1^{\text{PoK}}$





Verifier

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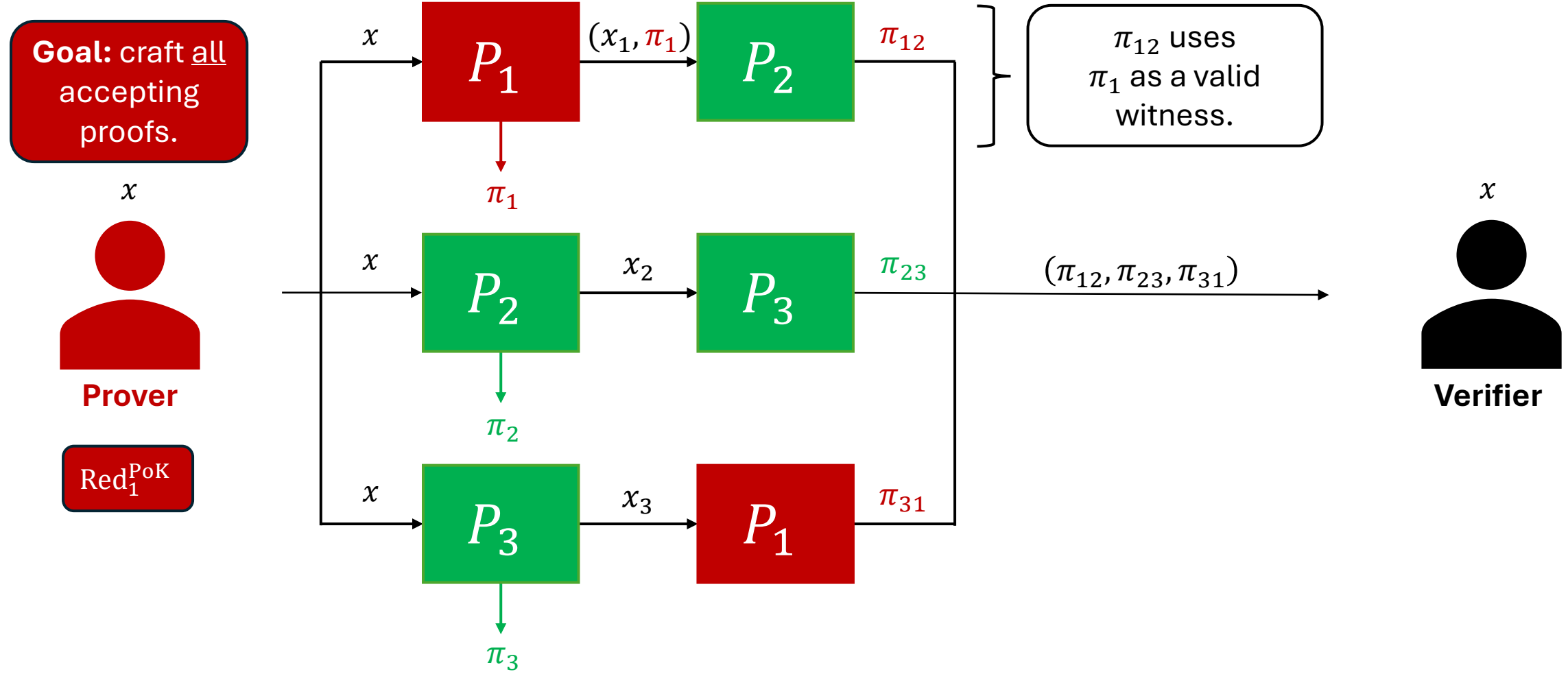

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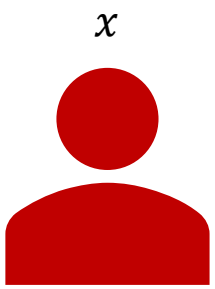

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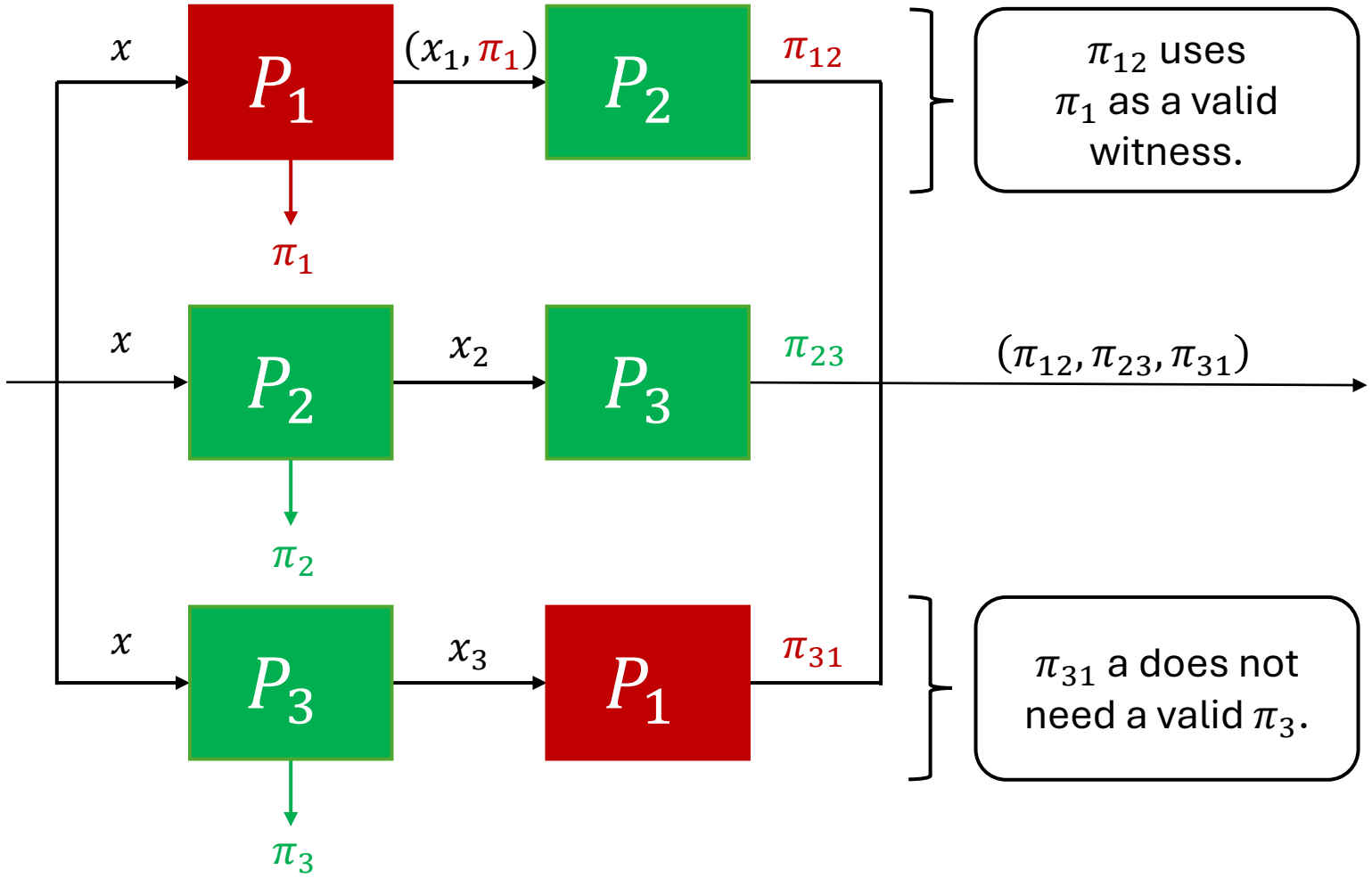
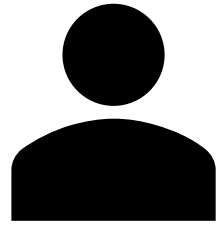


π_{12} uses π_1 as a valid witness.

$(\pi_{12}, \pi_{23}, \pi_{31})$

π_{31} a does not need a valid π_3.

x



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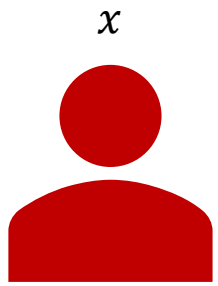
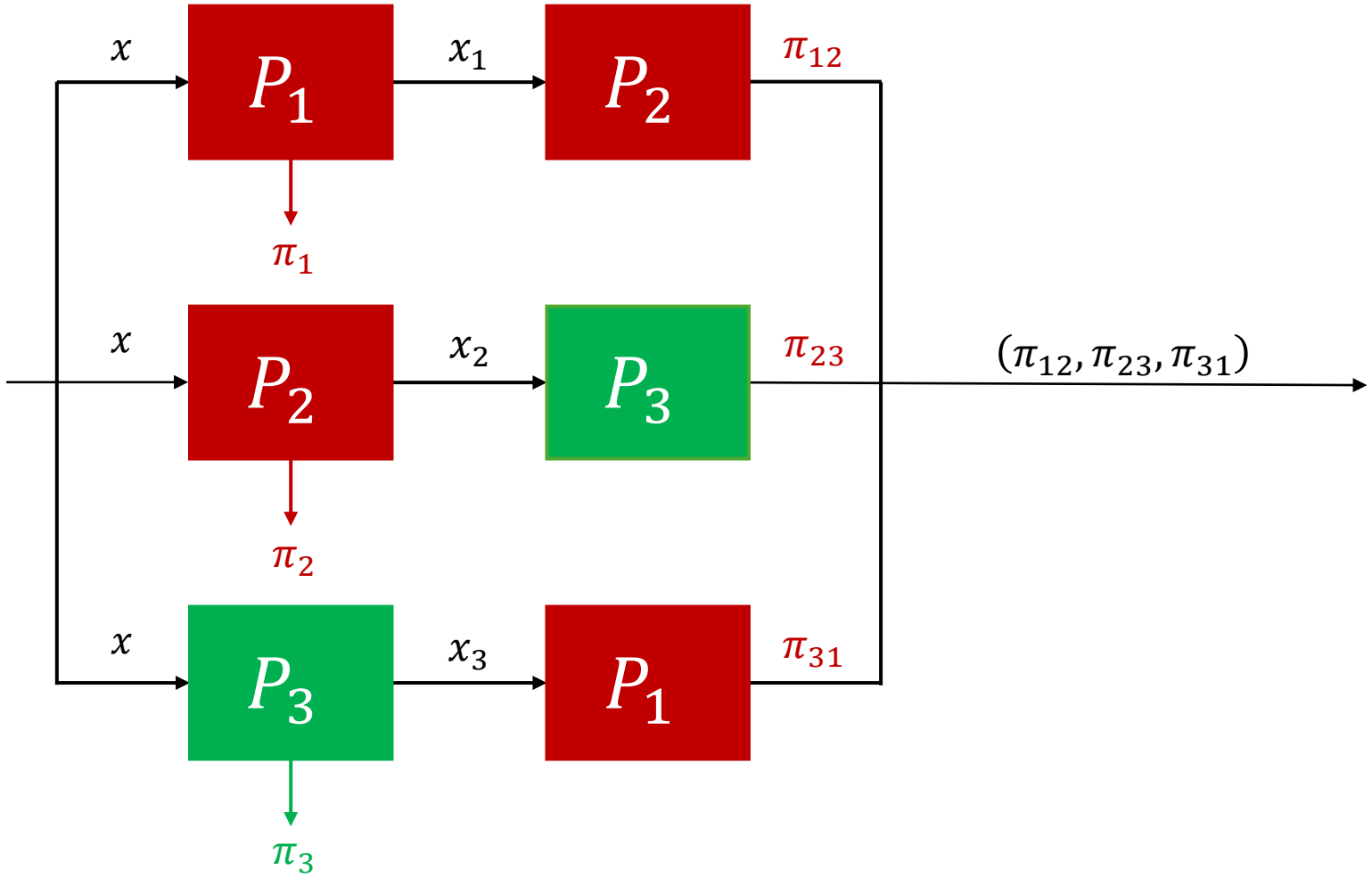


Prover

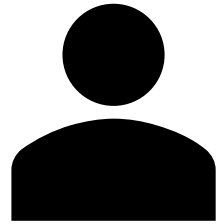
$\mathrm{Red}_1^{\mathrm{PoK}}$

$\mathrm{Red}_2^{\mathrm{PoK}}$

$(n - t) + 1 = 2$ reductions



x



Verifier

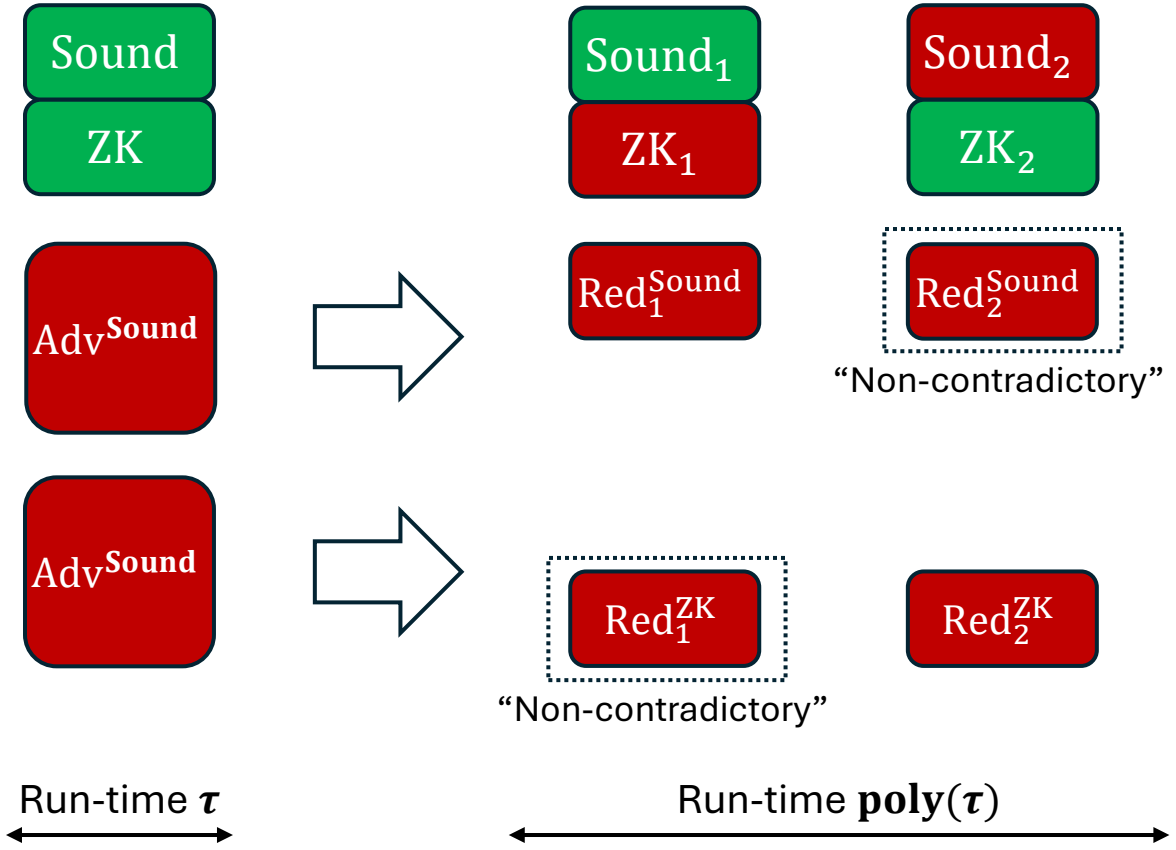
Backup Slides B

Impossibility Sketch

Impossibility for $t \leq n/2$

$n = 2$	$t = 1$
$(n - t) + 1 = 2$	

$$\mathrm{NIZK}_{\mathrm{comb}} = f(\mathrm{NIZK}_1, \mathrm{NIZK}_2)$$



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NIZK_1

- $\mathrm{Setup}(1^\lambda)$: Output $\perp$.
- $\mathrm{P}(x, w)$: Output w.
- $\mathrm{V}(x, \pi)$: Check w against x.

NIZK_2

- $\mathrm{Setup}(1^\lambda)$: Output $\perp$.
- $\mathrm{P}(x, w)$: Output 0.
- $\mathrm{V}(x, \pi)$: Output $\pi == 0$.

➢ NIZK_1 is **unconditionally** sound, NIZK_2 is **unconditionally** zero-knowledge.

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- NIZK_1 is **unconditionally** sound, NIZK_2 is **unconditionally** zero-knowledge.

- By contradiction, $\mathrm{NIZK}_{\mathrm{comb}}$ should output a **secure NIZK** upon input NIZK_1 and NIZK_2 as above.

Impossibility for $t \leq n/2$

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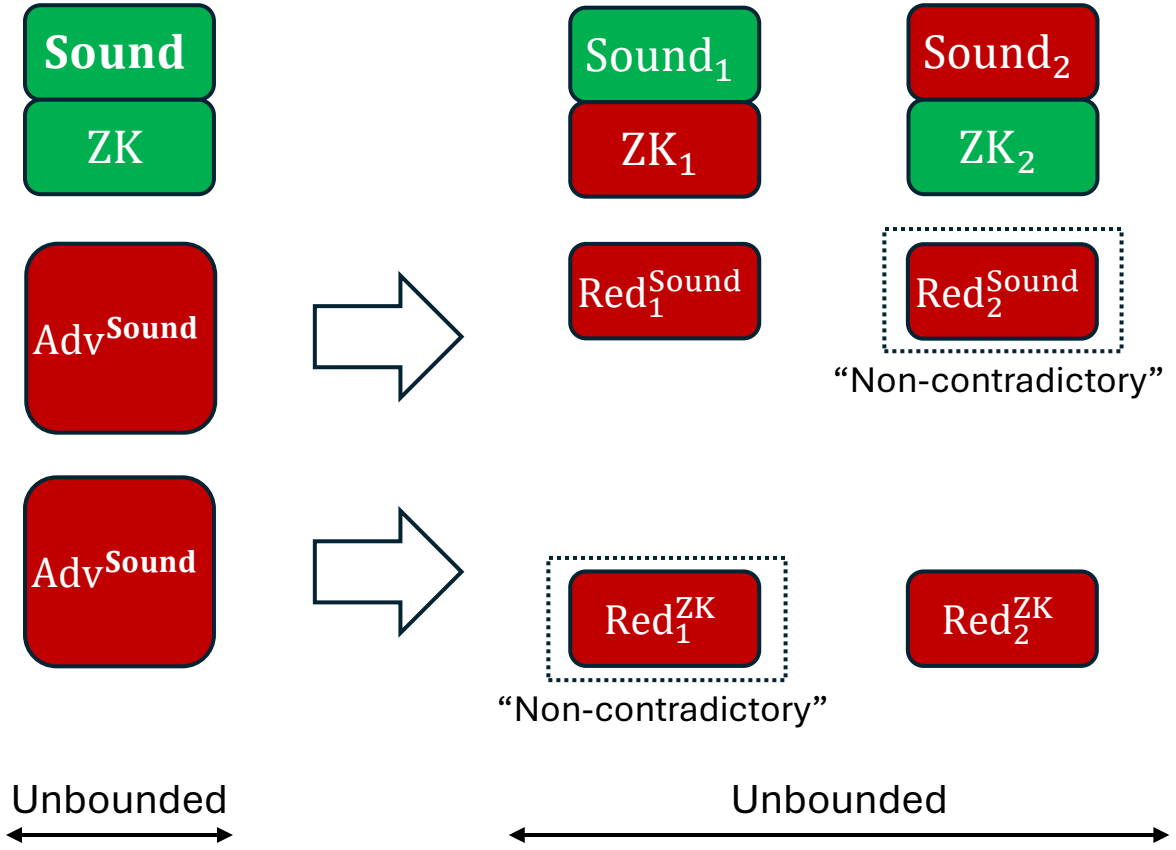

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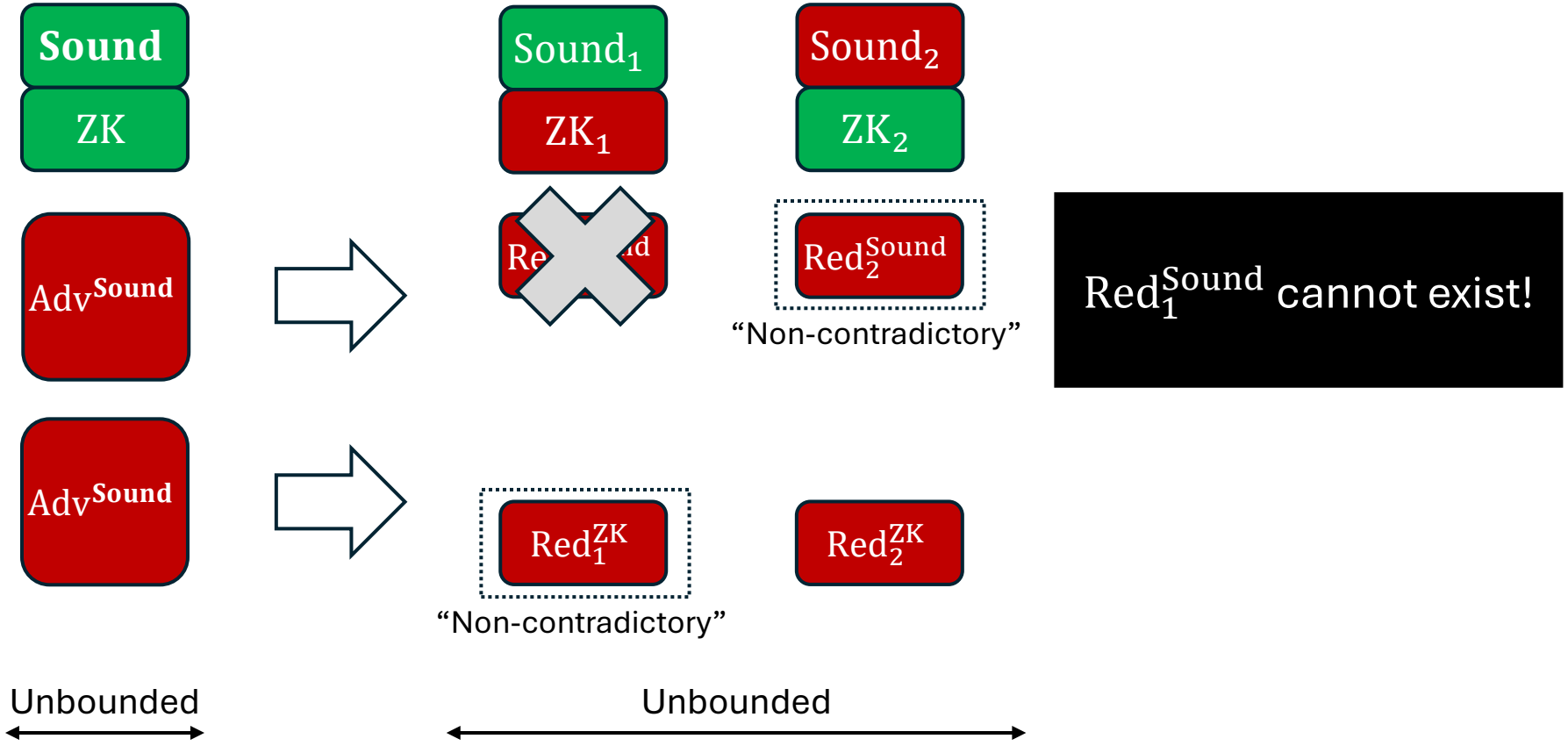

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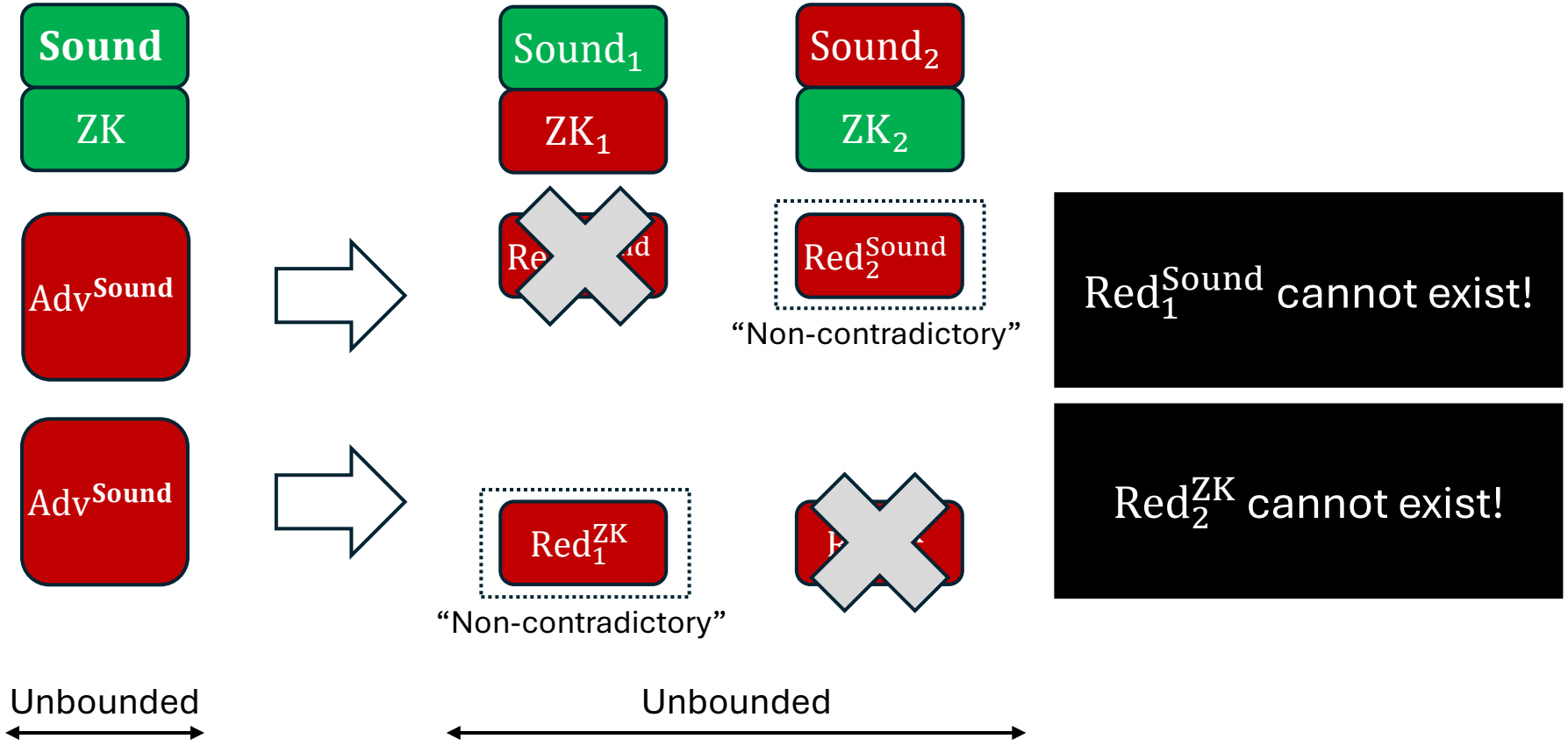

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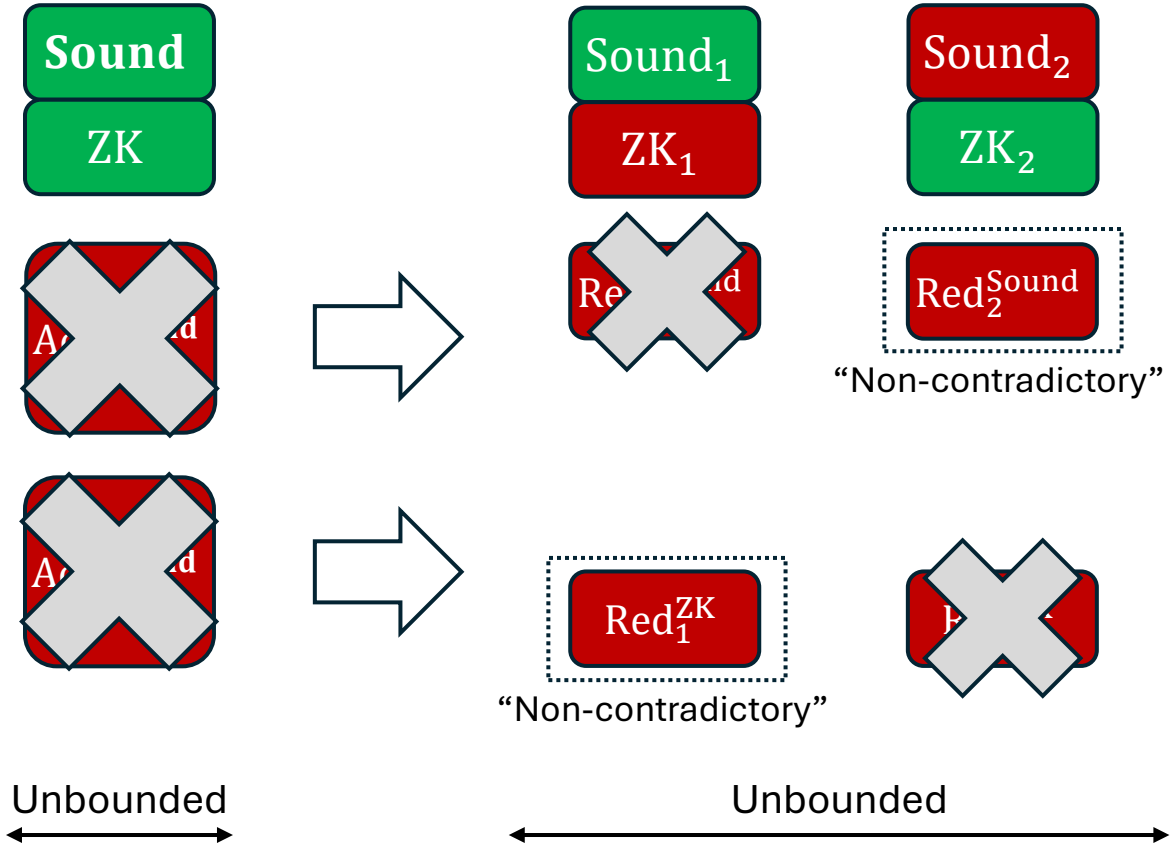

$\mathrm{Adv}^{\mathbf{Sound}}$ cannot exist!

$\mathrm{Red}_1^{\mathrm{Sound}}$ cannot exist!

$\mathrm{Red}_2^{\mathrm{ZK}}$ cannot exist!

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$\mathrm{NIZK}_{\mathrm{comb}}$ retains soundness even against unbounded adversaries.

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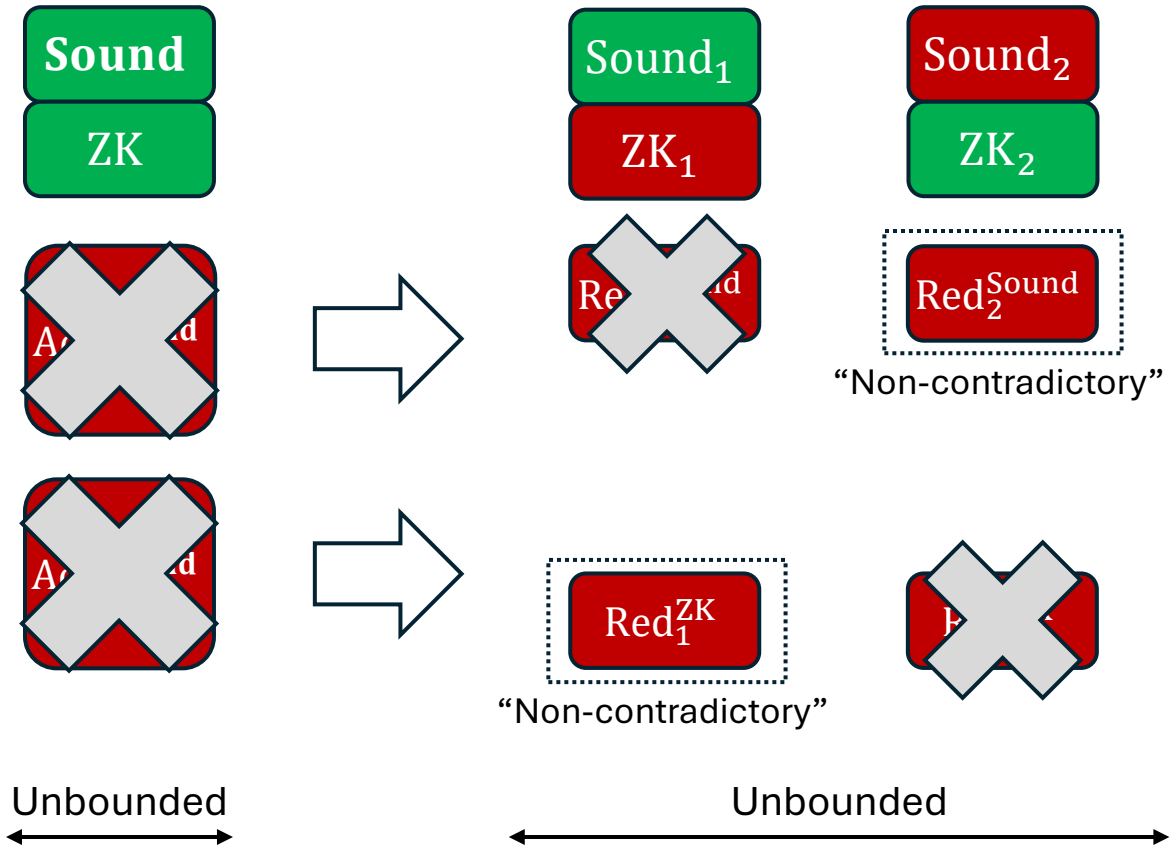

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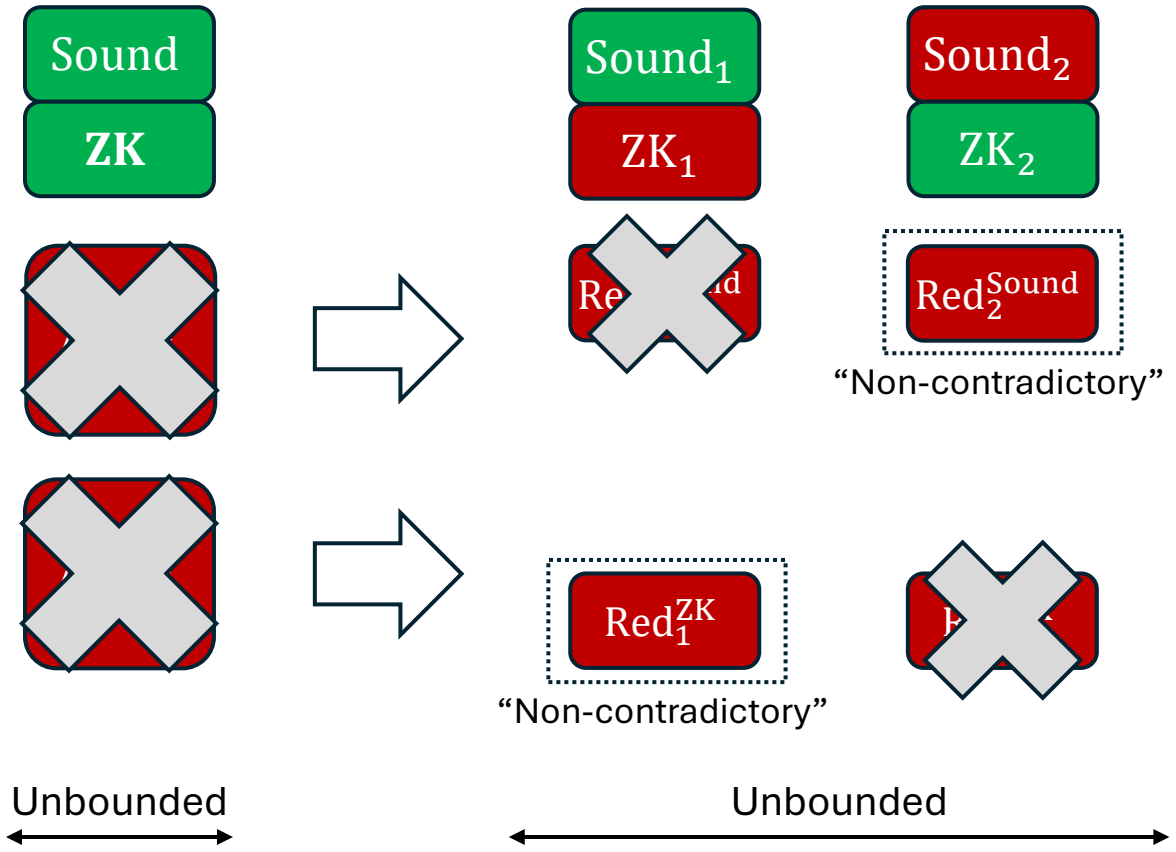

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$\mathrm{NIZK}_{\mathrm{comb}}$ retains zero-knowledge even against unbounded adversaries.

- $\mathrm{NIZK}_{\mathrm{comb}}$ is both unconditionally sound **and** unconditionally zero-knowledge.

[PS05] R. Pass, A. Shelat, "Unconditional characterizations of non-interactive zero-knowledge." CRYPTO'05. https://www.iacr.org/archive/crypto2005/36210114/36210114.pdf

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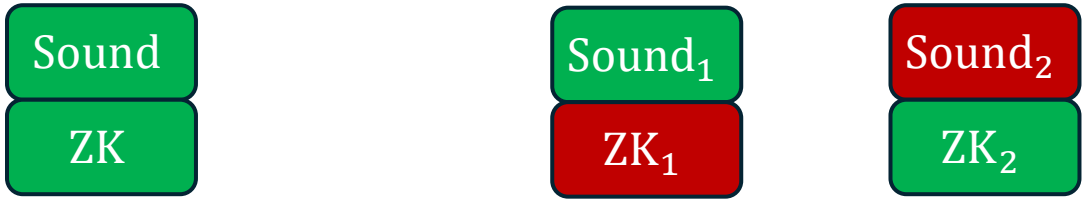

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- $\mathrm{NIZK}_{\mathrm{comb}}$ is both unconditionally sound **and** unconditionally zero-knowledge.
- Such a NIZK cannot exist for NP, unless the polynomial hierarchy collapses [PS05]!